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Controlling Information Premia by Repackaging Asset-Backed Securities

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ABSTRACT

Securities created from a base of underlying receivables are sold to uninformed "individual" and "institutional" hedgers. Institutions are more sophisticated than individuals because they are aware of the information-based transactions costs of all currently open markets and minimize transaction costs by trading in optimal portfolios. We show that profits earned from individuals can be optimized by changing the correlation coefficient between sets of receivables backing different securities, but profits earned from institutions are immune to changes in the correlation and can be controlled only by altering the number of securities created.

INTRODUCTION

Credit securitization began with the development of the residential mortgage-backed securities business in the 1960s and now includes products backed by a range of receivables, including commercial mortgages, auto loans and leases, home equity loans, student loans, credit card receivables, ticket sales, insurance premium loans, and an expanding list of other assets. Receivables are packaged, underwritten, and sold in the form of asset-backed securities, for many of which market-makers keep open markets. Buyers of these securities use them for hedging various risks. By the end of 1994, the value of outstanding securitized instruments exceeded $1.9 trillion, and more than $500 billion of securitization transactions were done in 1994 alone. Of the many benefits of securitization, perhaps the most important is its ability to increase the value of the receivables by selling them in higher quality and more liquid financial markets (Schwarcz, 1993; Hill, 1996).

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This article studies the optimal creation of securities from a pool of receivables when the goal of the issuer is to enhance the liquidity of these se-curities.

How many securities should be created from a given pool of receivables and what is the ideal statistical distribution between the receivables underlying the securities issued? The answer partly depends on how insiders with superior information about the receivables underlying the security might affect the liquidity of one asset versus multiple securities markets.[1] I follow Bagehot's (1971) intuition that market-makers compensate themselves for bad trades due to the adverse selection of insiders by making markets less liquid. This could shrink the base of customers trading in the markets and lower the profits from the sale. How illiquid these markets must be depends on the information content of the different securities issued, the composition of uninformed traders (individuals vs. institutions), and their understanding of linkages among the markets created.

In secondary markets securities backed by seemingly identical pools of receivables trade at different prices, reflecting the information content of receivables underlying the pools. Becketti and Morris (1991) and Stanton (1994) point out that much of the market commentary on mortgage-backed securities focuses on identifying so-called *fast pay* and *slow pay* pools—that is, pools that consistently pay out faster or slower, respectively, than apparently comparable pools. Stanton's study of the prepayment behavior of five 12 percent GNMA pools between January 1983 and December 1989 finds that the proportion of principal remaining relative to the original principal varies between 10 and 35 percent for the five pools. When the market lacks information on the prepayment characteristics of homeowners, pools are priced "generically," that is, all comparable GNMAs have roughly the same price at any point in time. In this situation, an investor who can identify fast and slow pay pools can profit from the market's inefficiency.

For most receivables securitized, it is possible to vary the correlation of different securities issued by the separation of microcharacteristics of these receivables. In the case of home mortgages, the prepayment behavior of homebuyers is to a large extent determined by the path of interest rates and other macroeconomic factors; but it also depends on other buyer characteristics, such as the number of children in the household and duration of marriage and job. Advances in computer and communications technologies have made possible the collection and dissemination of credit information on buyers. This role is often played by mortgage bankers, who often are also involved in the underwriting and repackaging of passthroughs. An agency with detailed information on buyer characteristics can create multiple securities by dividing the pool of buyers into different classes. The correlation between the securities created would depend on how the classes of buyers are chosen.

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[1] Ahimud and Mendelson (1986) provide an example of the importance of liquidity in total asset return. Consider a security whose holding period is two years, which is the historical average holding period of NYSE stocks. A trading cost of $0.04 in a $1 stock using an 8 percent discount rate implies a $0.04 + \frac{0.04}{1.08^2} + \frac{0.04}{1.08^4} + \ldots$ a 28 percent reduction in the net potential market value of the asset. Cutting trading costs to 2 percent would raise the market value by about 20 percent.

The optimal security design in the model depends on the nature of the client base to which the securities are sold. Securities are sold to two major classes of buyers: individuals and institutions. Both sorts of traders can use the asset-backed securities as part of an overall strategy to hedge endowment risks. It is assumed that, when only one asset is issued, the two sets of traders behave identically. However, when multiple securities are issued—each with the same correlation with the trader's endowment risks (hence offering the trader the same hedging possibilities) apart from differences in transactions costs—we shall assume that individuals use only one of the security markets available. They reduce their use of this security as the transactions costs of participating in this security market increase. Institutional holders will potentially use all available securities, optimizing on the transactions costs across the different markets.

Data from the 1992 *Survey of Consumer Finances* are used to motivate the assumption regarding the individuals owning one asset-backed security from the set that is available.[2] Of all respondents that owned any stock, 45 percent said they owned only one stock. Of these, roughly half said they owned stock in the company they worked in. Therefore, there is some evidence to show that families or "individuals" would choose only one security from the entire set available. This is just one puzzling aspect of individual behavior in securities markets; other anomalies and a review on the literature are available in Bertaut (1996).

Institutional traders capture the behavior of "large" institutional traders like insurance companies, investment banks, and mutual funds, who buy and sell asset-backed securities to match the hedging needs of their customers. The customers need to trade in asset-backed securities to hedge endowment risks, similar to those described for noise traders. Presented with the erratic liquidity needs of their customers, institutional traders optimally choose a portfolio from all open security markets so as to minimize transactions costs. For example, many mutual fund companies offer their customers a GNMA fund. The fund manager chooses the portfolio of GNMA securities on behalf of the customers. Further, they understand linkages between markets, in the sense of how order flow in one market affects prices in other markets.

The repackager must use different strategies to optimize profits earned from individuals and institutions. Because individuals ignore intermarket linkages, higher profits can be made off them by changing the per-security information content while keeping total information constant. This is done by repackaging the receivables and changing the correlation between the securities information. Institutional traders' costs are immune to such changes in security correlation. The costs of institutional traders depend on the number of securities with which they can form portfolios. With a larger number of securities to choose from, these trad-

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[2] The *Survey of Consumer Finances* is a triennial survey sponsored by the Federal Reserve with the cooperation of the Department of Treasury. It is designed to provide detailed information on U.S. families' balance sheets and their use of financial services, as well as on their pension rights, labor force participation, and demographic characteristics. The 1992 survey was conducted by the National Opinion Research Center at the University of Chicago, in which 3,906 families were interviewed. 1995 survey data are not yet publicly available.

ers buy and sell smaller quantities and incur smaller transactions costs while satisfying their hedging needs. Solutions for the optimal number of securities to be issued and the correlation between the receivables underlying these securities are provided as functions of (1) the objective of the security designer, which might be to maximize the profits earned off each kind of trader or to maximize the consumer surplus of hedgers; (2) the masses of each kind of trader in the market; and (3) the elasticity of increased hedging volume with respect to transactions costs.[3] All the parameters of the problem can be measured empirically by practitioners.

Substantial work has been done on the role of incomplete markets and the hedging opportunities created when existing assets are split up and sold. Allen and Gale (1989) characterize optimal securities that are used for spanning otherwise unhedged risks. Their conclusion is that "extremal" securities are optimal. They are extremal in the sense that, in every state, payoffs from the original assets are allocated to the security held by people that value it most. The spanning framework is a very useful one for modeling the splitting up and customizing of securities for specific needs. Investment banks on Wall Street explicitly create securities for very specific needs of large customers such as pension funds, which, due to the uniformity of their clients, might have a very idiosyncratic and large exposure to certain risks. However, the spanning properties of securities often must be sacrificed for liquidity reasons, which may be heavily impeded by the impact of asymmetric information between buyers and sellers. In this article, it is assumed that receivables underlying the asset-backed securities being marketed have some correlation with the endowments of hedgers. Each of the securities created potentially can be used to hedge endowment risks, but the sensitivity of the security return to changes in receivables varies across securities, depending on its information content. The goal here is to design the securities to control the transactions costs that arise as market-makers recoup their losses to insiders by charging transaction fees on every order.

Duffie and Jackson (1989) and Cuny (1993) study the design of futures contracts to maximize the profits of futures exchanges. Duffie and Jackson maximize T times volume, where T is an exogenous, technologically determined transaction cost per contract. In this article, the level of transactions costs is different for each security created, depending on its information content. In Cuny, the exchange charges investors an endogenously determined one-time entry fee for each contract. However, there are no per-unit costs of transactions thereafter. The assumptions of these articles are better suited for futures markets, where informational asymmetries are less of an issue than the asset-backed security market studied here.

Duffie and DeMarzo (1993) show that a model of security design based on asymmetric information and insider trade implies a convex (and hence option-like) objective function for the issuer. Based on insiders' superior information relative to the market, they have the option to sell the stock to the market or hold on to it—the familiar *lemons* phenomenon. Insiders signal information to outsiders by re-

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[3] Profit maximization may not be the only objective of the Federal Home Loan Mortgage Corporation. It may be for USAir and Chrysler, which have sold several pools backed by ticket sales receivables and automobile loans and leases, respectively.

taining different amounts of the security. Splitting an asset and giving the insider options on the parts increases the value for the insider. Duffie and Rahi (1995) survey other articles on security design within an asymmetric information framework. As in these articles, my model has a similar options mechanism to create value. However, it is assumed that the fraction of each pool of receivables retained by the insider is not observed by the market, ruling out signaling possibilities by the issuer. For example, it is well known that the three government-sponsored entities buying and selling mortgage-backed securities hold a substantial fraction of mortgages that they purchase. Although the value of their aggregate holdings are public information, the composition of their portfolio and changes in holdings of different pools are confidential. Additionally, the problem is studied with an endogenous determination of the volume and the composition of the uninformed customer base.

In a setting similar to the one here, Subrahmanyam (1991) provides conditions under which different stocks should be packaged together and sold as an index to lower transactions costs for liquidity traders. This article provides results for further gains from repackaging and splitting are provided. Boot and Thakor (1993) provide conditions under which splitting assets is optimal to provide incentives for information acquisition for some traders. In this article, there is no information acquisition, yet splitting is able to lower transactions costs.

The next section presents the major assumptions and the structure of the model. This is followed by a study of the existence and characterization of equilibrium. Then, securities are designed to control profits earned from individuals and institutions, respectively. Finally, the analysis is extended to markets where both sorts of buyers are present.

THE MODEL AND DISCUSSION OF ITS COMPONENTS

I consider an economy with three stylized periods indexed by the variable t. Agents are perfectly patient, so there is no time discount. At t = 0, N risky securities are traded. The securities will split up at t = 1 the payoff on a risky asset whose payoff is comprised of the sum of returns from a pool of receivables. The joint distribution of the sum of the receivables in the pool is given by

$$\tilde{S} = 1 + \tilde{\upsilon}, \qquad (1)$$

where $\tilde{\upsilon}$ is a normally distributed random variable with a mean of zero and variance σ_{υ}^2.[4] The asset is to be sold either as specified or repackaged into N securities $\tilde{S}_i$, where

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[4] It is well known that in Kyle-style insider trading models, insider profits do not depend on the mean of the insider's information. Only unexpected components of information are of value. To make the notation simple, I have set the mean of each security to $\frac{1}{N}$ and separated out the unexpected component.

$$\tilde{S}_i = \frac{1}{N} + \tilde{\upsilon}_i. \tag{2}$$

We will require that

$$\sum_{i=1}^{N} \tilde{S}_i = \tilde{S}.$$

I shall assume that $\tilde{\upsilon}_i$ have mean zero and variance σ_i^2, respectively, and, for technical convenience, the distributions are normal. The security structure is summarized in the variance-covariance matrix of signals Σ_v, which is constrained to satisfy

$$\mathbf{1}' \Sigma_v \mathbf{1} = \sigma_\upsilon^2. \tag{3}$$

At $t = 1$, settlement of claims on all traded securities are made. Traders then use their proceeds to purchase "durable" assets that provide returns to them at $t = 2$ (this shall be discussed below in the description of the traders).

It is assumed that the security designer can divide up the receivables into parts with any desired covariance matrix Σ_v. This may or may not be possible given the correlations among the receivables. Generally, the greater the heterogeneity among the receivables, the larger the set of security correlations (designs) the designer can choose from. The optimal security designs here are constrained by the available correlation set. Typically, the available set of correlations is a connected set (in the case of two securities, the available set of correlations is an interval), and the objective functions of agents (to be described below) are continuous functions of the correlation parameters. The constrained solution will be the point in the available set closest in distance to the unconstrained choice. The example below illustrates the possibilities.

Example 1

Receivables are one of two types; receivables in each set are perfectly positively correlated with each other and have a correlation of ρ with each receivable in the other set. Let the sum of receivables in each set be Y_1 and Y_2, respectively, each distributed normally with mean one and variance one. Consider the formation of two pools: $X_1 = \delta Y_1 + \beta Y_2$ and $X_2 = (1 - \delta) Y_1 + (1 - \beta) Y_2$, where $\delta, \beta \in [0,1]$. Then any correlation coefficient $\rho_{x_1 x_2}$ in the interval $[\rho, 1]$ can be created by choosing appropriate $\delta, \beta \in [0,1]$. It is easily verified that

$$\rho x_1 x_2 = \frac{(1-\beta)\beta + (1-\delta)\delta + (\beta(1-\delta) + (1-\beta)\delta)\rho}{\left((1-\beta)^2 + (1-\delta)^2 + 2(1-\beta)(1-\delta)\rho\right)^{0.5} (\beta^2 + \delta^2 + 2\beta\delta\rho)^{0.5}}.$$

In particular, with $\delta = \beta$, $\rho_{x_1 x_2} = 1$, and with $\delta = 1$, $\beta = 0$, $\rho_{x_1 x_2} = \rho$. Because the right-hand side of the above equation is continuous in δ and β, it is evident that all values between ρ and one can be obtained by appropriately adjusting the weights.

The model is based on the single-period insider trading model of Kyle (1985) and its extension to the multi-asset case by Caballe and Krishnan (1994). There are five types of agents: the original issuer/security designer, a monopolistic insider, a risk-neutral market-maker, a continuum of noise traders, and a finite number N of liquidity traders.

Description of the Agents

The joint distribution of the $\tilde{\upsilon}_i$ is chosen by the security designer. I shall assume that securities are designed to either maximize or minimize (depending on the exact institutional role of the security designer) the profits from trading earned from different classes of uninformed customers. The trades of different uninformed traders are not observed. However, expected profits from each class can be calculated. The insider will take the security design as given and maximize profits. The sharing of the surplus between the insider and the security designer is achieved at the time of sale of the relevant data set containing information on the receivables. In the case where the security designer wants to maximize profits also, the price of the information is determined by a bargaining process. Assuming that each player is equally patient and a repeated offers game is played before trading starts, each player gets half the total profits, as in Rubinstein (1982). Therefore, both players want to maximize profits. If the profits of issuing N securities are greater than the profits of issuing just the original asset, then the original asset is not released into the market. In such a case, there shall be no organized trading of this asset. If profits from issuing the original asset are greater, then no other securities are sold.

The monopolistic insider observes the values of $\tilde{\upsilon}_i$ before trading starts. The securities are designed by the security designer before the insider observes $\tilde{\upsilon}_i$.[5] The insider submits his orders along with the noise and liquidity traders. He makes profits on the N securities at the expense of noise and liquidity traders. The security designer will maximize or minimize the costs of different classes of uninformed traders (described below), depending on his institutional role. In the case where the objectives of the insider and the security designer are the same, this may be the same person.

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[5] Because the securities are designed before the insider observes the information components $\tilde{\upsilon}_i$, the profits of the insider will be the same, even if $\sum_{i=1}^{N} \tilde{S}_i \neq \tilde{S}$, as long as the securities created satisfy $\mathbf{1}' \Sigma_\upsilon \mathbf{1} = \sigma_\upsilon^2$. Becketti and Morris (1991), among others, point out that almost all mortgage pass-throughs are so-called modified pass-throughs. Investors are guaranteed to receive interest and principal payments even if mortgages are in default or delinquent. This is done by smoothing payments across different mortgages, which is possible as long as the sum of the securities issued has the same distribution as the original pool of assets.

The market-maker observes the order flows in all assets. As in Kyle (1985), the market-maker sets the price of each asset to the conditional expected value of the underlying payoff given each asset. The market-maker forms an expectation based on knowing the distribution of noise and liquidity trade, the distribution of the underlying asset, and the equilibrium strategy of the insider in each security.

There are two sorts of uninformed players: a continuum of "noise" traders with mass γ_z, representative of the behavior of "small" traders or individuals and a continuum of "liquidity" traders with mass γ_y, representative of the behavior of "large" traders or institutions. Each type of trader behaves identically when there is only one marketed security. At t = 0, each trader j has a unit of an endowment security (security E) that can be either consumed or kept for one period. At t = 1, security E provides a random return of $1 + \tilde{\varepsilon}_j$. $\tilde{\varepsilon}_j$ is identically and normally distributed for each individual with a mean of zero and variance of σ_υ^2 (the same as the variance of return on the asset-backed security). Further, $\rho_{\varepsilon,\upsilon} = -1$. Therefore, one unit of the (unsplit) asset-backed security provides a perfect hedge to one unit of security E. Each unit of security E has a price of one at t = 0. At t = 1, the trader needs one unit of the single good in the economy to purchase a "durable" that will provide a sure return of $1 + \tilde{r}_j$ at time t = 2. $\tilde{r}_j$ is known to the trader; across traders, $\tilde{r}_j$ is distributed with distribution function F(r).[6] The trader is infinitely risk averse with respect to consumption at t = 2 and is unable to borrow at t = 1. Therefore, to ensure the purchase of the durable, the trader must hedge the endowment risk by selling half a unit of the endowment asset and purchasing half a unit of the asset-backed security at time t = 0.

The trader must place the order before observing the price of the marketed security; the trader's decision on the quantity to purchase is made assuming that the security will be purchased at the expected price given the market-maker's rule. Given the market-maker's rule, the expected price of the asset-backed security equals one. At t = 1, the trader also pays a transaction cost of λ per unit of the asset bought or sold. If the agent buys z units of the asset, the final return at t = 2 equals

$$w_j = (1 - z)(1 + \varepsilon_j) + z(1 + \tilde{\upsilon}) + \mathbf{I}_{\{\mathrm{var}[w_j]=0,\, \tilde{r}_j - z\lambda \geq 0\}}(\tilde{r}_j - z\lambda),$$

where **I** denotes the indicator function. The last term is the additional return realized at t = 2 from purchase of the durable, if trader j is to obtain a riskless return of 1 at t = 1, and if the benefit $\tilde{r}_j$ exceeds the transactions costs of purchasing the asset-backed security. Because the trader is infinitely risk averse with respect to consumption at t = 2, he or she plans to purchase only the durable if it is possible to reduce the variance of the return to zero. Therefore, if transactions costs are lower than the benefit, each trader will hold half a unit of the asset for every unit of en-

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[6] The durable might be, for example, a physical asset such as a house or investment in education for siblings.

dowment in period zero to drop the variance of period one's return to zero. If transactions costs λ exceed benefit of the durable $\tilde{r}_j$, trader j will not enter the security market. Trader j's optimal demand for the asset is, therefore,

$$\tilde{z}_j = 0.5 \quad \text{if} \quad \tilde{r}_j > 0.5\,\lambda \text{ and} \tag{4}$$

$$\tilde{z}_j = 0 \quad \text{if} \quad \tilde{r}_j \le 0.5\,\lambda. \tag{5}$$

The number of agents long in security E at time t = 0 is a normally distributed random variable $\tilde{e}_0$, with mean zero and variance $\gamma_y + \gamma_z$.[7] $\tilde{e}_0$ and $\tilde{r}_j$ are independent for all j. Therefore, the aggregate volume of the asset-backed security is distributed normally with a mean of zero and a variance $0.25\,((1 - F(0.5\,\lambda))^2\,(\gamma_y + \gamma_z)$. Throughout this article, it is assumed that the distribution function F(r) is continuous. Since 1 − F(r) plays the role of the demand curve of traders with respect to transactions costs, this assumption ensures that there is a unique level of transactions costs that will maximize the profits of the security designer. To get closed-form solutions that will further aid our intuition, explicit solutions will be worked out for the case where F(r) has a uniform distribution function on [0,M], where M is an upper bound for r.

Since the number of traders with endowment risks is random, the aggregate trade in the distribution has a distribution similar to those of noise traders in Kyle (1985). The costs of trading in a market are measured by the price sensitivity of demand to order flow.[8] It measures the cost of turning around a position. The aggregate demand exhibits two important properties: The demand for the security is proportional to its variance, because the security is used for hedging. And the variance of the demand is inversely related to transactions costs because use of the asset-backed security declines when transactions costs increase.

When only the single asset is issued, the behavior of the two sets of uninformed players is identical. When multiple securities are issued, we make different behavioral assumptions of the two kinds of players. Their demands are described below. Let Γ_{ii} be the increase in price of the ith asset in the market when the order flow in the ith market increases by one unit.[9]

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[7] The normal distribution is simply for technical convenience; the interpretation is that a negative number of agents implies that, in aggregate, agents have sold security E short and received the price of one at t = 0 for the short sale. To hedge their risk, agents with short sales of E must sell the asset-backed security.

[8] It is useful to recall that, in the Kyle model, there is no bid-ask spread—a more standard measure of transactions costs.

[9] The Γ matrix is potentially a function of ω, the information of the market-maker. The results in Caballe and Krishnan (1994) imply that the market-makers' strategy can be written as a linear rule $P(\tilde{\omega}) = P_0 + \Gamma\tilde{\omega}$, where $\tilde{\omega}$ is an $N \times 1$ vector of order flows, and Γ is an $N \times N$ constant matrix of price sensitivities to the order flow.

Noise (Individual) Trader's Demand for the Securities

Suppose the security designer creates N securities. Each trader randomly decides to hedge endowment risk in one of the available security markets. Since each of the securities has the same correlation with the endowment, this is not an irrational assumption from a hedger's perspective. However, the trader does not compare transactions costs on open security markets when completing the trade. Each trader also has an expectation of the information content σ_i^{2e} of the security. In equilibrium, the trader's expectation equals the action of the security designer, that is, $\sigma_i^2 = \sigma_i^{2e}$. Suppose that $\sigma_i^2 = K\sigma_\upsilon^2$. The aggregate demands of these noise traders are given in Lemma 1 below.

Lemma 1

Let the variance of the ith security be given by σ_i^2, and r_j is distributed with distribution function F(r) = 1/M for $0 \le r \le M$. To hedge endowment risk, each noise trader demands z_{ji} units of the asset-backed security per unit of endowment. Under the assumption that each security has perfectly negative correlation with the endowment, the trader optimally demands z_{ji}, which equals

$$\begin{aligned} \tilde{z}_{ji} &= \frac{1}{\frac{1}{N} + \sqrt{K}} \qquad \text{if} \quad \tilde{r}_j > \tilde{z}_{ji}\,\Gamma_{ii} \quad \text{and} \\ \tilde{z}_{ji} &= 0 \qquad\qquad\quad \text{if} \quad \tilde{r}_j \le \tilde{z}_{ji}\,\Gamma_{ii} \end{aligned} \tag{6}$$

for each trader j, aggregate noise traders' demand in the ith security is normally distributed with a mean of zero and variance equal to

$$\left(\frac{1}{\frac{1}{N} + \sqrt{K}}\right)^2 \left(1 - \frac{z_{ji}\,\Gamma_{ii}}{M}\right)^2 \frac{\gamma_z}{N}.$$

Proof

See the Appendix. Note that, when the single asset is issued, then N = 1 and K = 1 and, therefore, the same demand as in equations (4) and (5) is obtained.

Liquidity (Institutional) Trader's Demand for the Securities

These traders capture the behavior of institutional traders like investment banks and mutual funds, who frequently buy and sell asset-backed securities to match the hedging needs of their customers. The customers need to trade in asset-backed securities to hedge endowment risks, similar to those described for noise traders. Presented with the erratic liquidity needs of their customers, institutional traders optimally choose a portfolio from all open security markets so as to minimize transactions costs. Although these traders are not necessarily better informed about the payoffs of assets than noise traders, they have better information on the depth and transactions costs of the various markets that are open. Since the market-maker sets prices conditional on observing the order flow in all securities, they take into account how order flow in one security affects the prices of all other securities.

The liquidity trader choose a portfolio α for each dollar of endowment risk. Given the endowment risk, the trader will seek to minimize $C_y(\Gamma,\alpha) = \alpha^{\mathbf{T}}\, \Gamma\, \alpha$ subject to the constraint that $\alpha^{T}\, \Sigma_{u}\, \alpha = \sigma_{u}^{2}$. The constraint implies that the portfolio will provide the hedger with exactly the same hedging possibilities as when there is a single asset, albeit with the transactions costs $C_y(\Gamma,\alpha)$. Therefore, the liquidity trader optimally demands

$$\tilde{z}_{j,i} = \alpha_i\, 0.5 \qquad \text{if} \quad \tilde{r} > C_y(\Gamma,\alpha) \quad \text{and} \tag{7}$$

$$\tilde{z}_{j,i} = 0 \qquad \text{if} \quad \tilde{r} \le C_y(\Gamma,\alpha) \tag{8}$$

for i = 1, ..., N. In the special case when equally informative securities are issued and $\Gamma_{11} = \Gamma_{22}$, it is evident that $\alpha_i = 1$ is optimal for each liquidity trader.

Let Σ_y and Σ_z be the variance-covariance matrices of liquidity traders' and noise traders' demands for the two securities, respectively. Then, by Lemma 1,

$$\Sigma_z = \begin{bmatrix} z_{ji}^2 (1 - F(z_{ji}\Gamma_{11}))^2 \frac{\gamma_z}{N} & 0 & 0 & 0 & 0 \\ 0 & z_{ji}^2 (1 - F(z_{ji}\Gamma_{22}))^2 \frac{\gamma_z}{N} & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & z_{ji}^2 (1 - F(z_{ji}\Gamma_{NN}))^2 \frac{\gamma_z}{N} \end{bmatrix}. \tag{9}$$

And, by equations (7) and (8),

$$\Sigma_y = 0.25 \begin{bmatrix} \alpha_1^2 & \alpha_1\alpha_2 & \alpha_1\alpha_3 & \cdots\alpha_1\alpha_N \\ \alpha_2\alpha_1 & \alpha_2^2 & \alpha_2\alpha_3 & \cdots\alpha_2\alpha_N \\ \dots & \dots & \dots & \dots \\ \alpha_N\alpha_1 & \alpha_N\alpha_2 & \alpha_N\alpha_3 & \cdots\alpha_N^2 \end{bmatrix} (1 - F(C_y(\Gamma))^2\, \gamma_y. \tag{10}$$

Let $\Sigma = \Sigma_y + \Sigma_z$. I shall henceforth call equations (9) and (10) the "volume equations," because second moments of trade are a measure of the round-trip transactions of noise and liquidity trade in the N securities.

In sum, an explicit hedging objective has been made for each trader, and hedging volumes have been derived. It is assumed that the traders are not perfectly rational; traders anticipte completing their transactions at the expected price of the security, given the market-maker's pricing rule. It is not possible to have perfectly rational agents and yet obtain endogenously derived transactions costs due to insider trading. This is due to the well known "no-trade" theorem (see, e.g., Milgrom and Stokey, 1982) that implies no insider trade in equilibrium when all agents are perfectly rational. The traders do, however, correctly anticipate the slope of the market-maker's pricing schedule, which determines the transactions costs, and reduce their hedging needs as transactions costs decline.

EQUILIBRIUM

Let $\tilde{X}_i$, $\tilde{Y}_i$, and $\tilde{Z}_i$ be the trades of the insider, the liquidity traders, and the noise traders in security i. The same letters with i subscripts refer to the N × 1 vectors of the same variables, representing the above quantities in each security. Let $\tilde{\omega} = \tilde{X} + \tilde{Y} + \tilde{Z}$ be the total order flow vector for the N securities. Profits of the informed trader in security i are given by $\pi_i = (S_i - P_i)\, X_i$. Let $\tilde{Y}_i = \alpha_i \tilde{Y}$ be the liquidity trader's trade in the ith security.[10] I analyze equilibrium of the following form:

Profit maximization. The insider's strategy $X(\tilde{\upsilon})$ satisfies

$$\sum_{i=1}^{N} E[\pi_i(X(\tilde{\upsilon}), \tilde{Y}, \tilde{Z}, P(\tilde{\omega}))] > \sum_{i=1}^{N} E[\pi_i(\hat{X}(\tilde{\upsilon}), \tilde{Y}, \tilde{Z}, P(\tilde{\omega}))] \tag{11}$$

for any other strategy $\hat{X}(\tilde{\upsilon})$.

Semi-strong market efficiency. The pricing rule $P(\omega)$ satisfies

$$P(\omega) = E[\tilde{S} \mid \tilde{\omega}]. \tag{12}$$

Optimizing portfolio choices. Noise traders buy securities to completely hedge their endowment risks as given by Lemma 1. The portfolio choice of liquidity traders are given by equations (7) and (8). α minimizes $\alpha\ \Gamma(\tilde{\omega})\ \alpha^T$ subject to the constraint that $\alpha^T \Sigma_\upsilon \alpha = \sigma_\upsilon^2$. Traders have expectations about the information content of the matrix Σ_υ^e. In equilibrium, these expectations are correct.

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[10] All liquidity traders make the same portfolio choices, and, hence, I avoid subscripts for each trader.

Optimization of informed trader's costs. The security designer maximizes

$$\theta\, \Gamma\, \Sigma_z + \upsilon\, \Gamma\, \Sigma_y,$$

where θ and υ are both in $\{-1,1\}$. The first term in the summation represents the equilibrium costs of noise traders (individuals), and the second term represents the costs of liquidity traders (institutional traders). The weights given to the two classes θ and υ vary with the role of the security designer. A negative weight implies that the security designer wants to minimize the costs of that class of uninformed agents.

The equilibrium notion is not quite a game theoretic one because market-makers and noise traders do not explicitly maximize any particular objective. Typically, the market efficiency condition which implies zero expected profits has been justified by invoking Bertrand competition among risk-neutral market-makers. Dennert (1993) has shown that the condition obtains if it is assumed that each market-maker can observe the aggregate order flow as opposed to only the order flow directed to him, and that ties are broken by equally splitting the aggregate order flow.

I now provide an explicit characterization of a linear equilibrium. The characterization is an extension of Proposition 3.1 in Caballe and Krishnan (1994).

Result 1

There always exists an equilibrium defined as follows: The price function is

$$P(\tilde{\omega}) = 0.5 + \Gamma\, \tilde{\omega},$$

where

$$\Gamma = \frac{1}{2} \Sigma^{-0.5}\, M^{0.5}\, \Sigma^{-0.5},$$

and

$$M = \Sigma^{0.5}\, \Sigma_\upsilon\, \Sigma^{0.5}.$$

The strategy of the insider is

$$X(\tilde{\upsilon}) = \Theta\, \tilde{\upsilon},$$

where $\Theta = \Gamma^{-1}$.

This is the unique linear equilibrium.

Comment 1

The unique positive definite square root of the symmetric positive definite matrix M is given by

$$M^{0.5} = E\Lambda^{0.5}E^{T},$$

where $\Lambda^{0.5}$ is a diagonal matrix with the positive square roots of the eigenvalues of M along the diagonal, and the corresponding orthogonal eigenvectors are the columns of the matrix E. Like the square root of a number, $M^{0.5}M^{0.5} = M$.

Comment 2

The proof of existence of equilibrium is similar to that of the single period equilibrium in Kyle (1985). Caballe and Krishnan (1994) prove existence for the multi-asset case. Bhushan (1991) shows that having liquidity traders simply requires an adjustment to the Σ matrix to account for the covariance in order flows which liquidity trade induces. The extension I provide here is for the case where the variance of noise and liquidity trade may depend on the price sensitivity paramaters set by the market-maker. To illustrate the point in as simple a framework as possible, I show the necessary steps in the single asset case with only noise traders.

The insider's profits are written as $E\left[[\tilde{S} - P(\tilde{\omega})]\tilde{X}|\tilde{X}(\tilde{\upsilon})\right.$, where $\tilde{\omega} = \tilde{X} + \tilde{Y} + \tilde{Z}$. Suppose the insider conjectures that the market-maker's rule is linear in the order flow $P(\tilde{\omega}) = 1 + \lambda\omega$. As in Kyle, profits are quadratic in $\tilde{X}$; hence, the insider's strategy is linear in its signal and can be written in the form X $= \beta\tilde{\upsilon}$, where $\beta = \frac{1}{2\lambda}$. The market-maker sets $P(\tilde{\omega}) = E[\tilde{S}|\tilde{\omega}]$. By the projection theorem and normality of order flow and noise trade, $P(\tilde{\omega}) = 1 + \lambda\omega$, where λ satisfies

$$\lambda = \frac{\beta\sigma_{\upsilon}^{2}}{\beta^{2}\sigma_{\upsilon}^{2} + \gamma_{z}(1 - F(\lambda))^{2}}. \tag{13}$$

This verifies the insider's conjecture that the market-maker follows a linear strategy. Equilibrium will therefore exist as long as there is a real finite positive solution to equation (13). This will be true in general if the distribution function is continuous. Equilibrium may exist for some parameter values even if there are jumps. In this article, for convenience, I will choose a uniform distribution function.

Comment 3

In equilibrium, the sum of expected profits of the insider in the two security markets can be written as $\mathrm{Tr}(\Gamma\,\Sigma_y) + \mathrm{Tr}(\Gamma\,\Sigma_z)$, where the first component is the cost incurred by liquidity traders and the second by noise traders. As is evident from the volume equation (9), the costs to noise traders in each market depend only on own price sensitivities to order flow Γ_{ii}. The cost to liquidity traders as is evident from equation (10) in addition to those depends on cross price sensitivities Γ_{ij}. Γ_{ij} represents a change in the price of the ith security due to a unit change in order flow in the jth security market. Therefore, $\alpha\ \Gamma(\tilde{\omega})\ \alpha^T$ measures the change in the total costs of hedging, including, in addition to the own price effects on trading, the effects trades in security i have on prices of all other securities $j \neq i$ (measured by the off-diagonal elements Γ_{ij}).

Comment 4

The results in Kyle (1985) can be extended to show that the market-maker's rule can be written as $P(\omega_t) = P_0 + \lambda\,\omega_t$, where λ is the solution to the equation

$$\lambda = \frac{1}{2}\left(\frac{\sigma_\upsilon^2}{\gamma_y \sigma_y^2(\lambda) + \gamma_z\,(1 - F(\lambda))^2}\right)^{0.5}, \tag{14}$$

and expected profits can be written as $\lambda\,(\gamma_z + \gamma_y)\,(1 - F(\lambda))$. The single asset profits put a lower bound on the profits which the security designer can ensure for the insider with appropriate security design. Note that, with a single security, there is no distinction between noise and liquidity traders. The noise traders in Kyle (1985) would have $1 - F(\lambda) \equiv 1$. In this case, $\lambda = 0.5\left(\sigma_\upsilon^2 / (\gamma_y + \gamma_z)\right)^{0.5}$, and expected profits of the insider equal $0.5\left(\sigma_\upsilon^2\,(\gamma_y + \gamma_z)\right)^{0.5}$.

Comment 5

As noted above, the Kyle (1985) model does not yield a bid-ask spread, a more standard measure of transactions costs relative to price elasticity. Krishnan (1992) provides an interesting equivalence between the Kyle and Glosten and Milgrom (1985) models. The latter model has a risk-neutral market-maker who sequentially services the trades of customers. Some of the customers might be informed about security payoffs while others are like noise traders in Kyle's model. The equilibrium strategy for the market-maker is to charge a bid-ask spread which compensates him in expected terms for each trade. In a binary version of Kyle's model, there is an equivalence in extensive forms for the two games. The slope (price elasticity) set by the market-maker in Kyle's model is identical to the bid-ask spread in the Glosten and Milgrom model.

SALE TO INDIVIDUALS

This section examines optimal security design when there are only indivduals in the market. It is shown that, with $N \geq 2$ securities, globally maximum profits can be attained—by adjusting the correlation of receivables backing the different securities. If securities are designed to maximize the consumer surplus of these traders (recall that these traders have a downward sloping hedging schedule), then the surplus continues to increase as a larger number of perfectly positively correlated securities are created. The globally maximum surplus is reached in the limit, as the number of securities issued approaches infinity.

Result 2

Let $1 - F(r) = (1 - \frac{r}{M})$ for $0 \leq r \leq M$ and $\gamma_y = 0$. Then

(i) Informationally unconstrained profits of the insider do not depend on the number of securities issued.

(ii) When the single asset is issued, the insider's profits will be generically lower than informationally unconstrained profits.

(iii) For any given number $N \geq 2$, the security designer can ensure the informationally unconstrained profits by issuing N equally informative securities and adjusting the correlation between their payoffs to satisfy

$$\left[1 + \frac{(N-1)(1-\rho)^{0.5}}{(1+(N-1)\rho)^{0.5}}\right] = \frac{M}{8}\frac{\gamma_z^{0.5}}{\sigma_\upsilon} N. \tag{15}$$

Proof

See the Appendix.

Note that the left-hand side in statement (iii) is a monotonic function in ρ for every N, so a unique solution exists for a given set of parameters on the right-hand side.

The main ideas underlying the proof are quite simple; the security designer is to be viewed as a price-setting monopolist facing a downward sloping hedging schedule with respect to transactions costs. Because each noise trader trades only in one market, his cost depends only on Γ_{ii}, the own-price elasticity of order flow in security i. To attain informationally unconstrained profits, the designer should set transactions of a trader in market i—$z_{ji}\,\Gamma_{ii}$ in the model—to M/3. Given z_{ji} in Lemma 1, one can back out the required depth of the market Γ_{ii}. The information content of security i, $\sigma_i^2 = \frac{1}{N + N(N-1)\rho}$, which is increasing in ρ for any N. Therefore, increasing ρ increases Γ_{ii}. The demand in Lemma 1 implies that increasing ρ lowers the quantity traded; however, the pricing rule of the market-maker implies that $z_{ji}\,\Gamma_{ii}$ is increasing in ρ. Therefore, the transactions costs of noise traders can be *controlled* by changing ρ for any $N \geq 2$. The optimal correlation calculated in Result 2 ensures the optimal level of transactions costs in each market. With the single asset, the security designer does not have this degree of freedom and hence cannot ensure informationally unconstrained profits.

The optimum can be attained for an N, although from equation (2) it is evident that, for a larger N, a smaller $|\rho|$ is needed. This is because the slope of the left-hand side with respect to ρ is increasing in N. The intuition behind this is as follows: with a larger number of securities, each market is thinner, and this tends to increase Γ_{ii}. To achieve the optimum level of transactions costs, the information content must be lowered by decreasing ρ as explained in the previous paragraph.

The consumer surplus of noise traders is maximized by driving the transactions costs of these traders down to zero. This ensures the largest probability of the hedger being able to adopt this strategy. The following result shows that, if the original asset is split up repeatedly into parts with uncorrelated receivables, then the aggregate costs of noise traders remain constant. However, if the securities have receivables which are perfectly positively correlated, then the aggregate costs of liquidity traders decrease to zero as the number of securities created increases.

Result 3

Let $1 - F(r) = \left(1 - \frac{r}{M}\right)$ for $0 \leq r \leq M$, and suppose $\gamma_y = 0$. Suppose the security designer issues N equally informative securities with correlation ρ between them. Then

(i) If $\rho = 0$, then the hedging cost in each security market, and, hence, the consumer surplus of traders, does not depend on N.

(ii) If $\rho = 1$, then the transactions costs in each market tend to zero, and N increases to infinity. Thus, surplus is maximized in the limit by issuing an arbitrary large number of equally informative and perfectly correlated securities.

Proof

See the Appendix.

(i) One provides an important consistency check on the model. With uncorrelated assets, the variance per security $\sigma_i^2 = \frac{\sigma_\upsilon^2}{N}$, an Nth of the variance of the single asset. Also, the mass of agents per security equals $\frac{\gamma_z}{N}$, an Nth of the total mass. Therefore, the signal-to-noise ratio in each market, measured by the information content of the security divided by the mass of noise trade, does not depend on N. Consequently, the market-maker charges the same transactions costs per unit of hedging need irrespective of N. When perfectly positively correlated securities are issued, the information content per security equals

$\sigma_i^2 = \frac{\sigma_u^2}{N + 2N(n-1)}$. This clearly decreases at a rate faster than N, while the mass of noise trade declines at a rate of N. Therefore, as N increases, the signal-to-noise ratio in each market declines, and the transactions costs decline to zero as N increases to infinity. This explains statement (ii). If there is a finite cost to issuing a new security, then, clearly, this will put a limit on the number of securities issued.

SALE TO INSTITUTIONS

Institutional traders, as defined above, observe transactions costs and optimize in all open markets. They counter any change in security design by appropriately adjusting their portfolios, making it extremely difficult to earn profits from them. It is shown that the only way to increase profits from these traders is to provide them with lower per-unit transactions costs and, hence, induce a larger volume of trade.

The first result is similar to those in Admati and Pfleiderer (1988) and Bhushan (1991). It shows that, if there is no noise trade, then liquidity traders find it beneficial to trade in a single security. Each trader benefits from trading solely in the thickest market available, and hence, as a group, the traders coordinate on one market. Some amount of noise trade in each security market would provide enough thickness for liquidity traders to enter these markets.

Result 4 (The Need for Some Noise Trade)

Suppose $\gamma_z = 0$; that is, the volume of noise trade is zero. Then liquidity traders trade in a single security. There is a breakdown of the market in other securities.

Proof

If $\gamma_z = 0$, $\Sigma = \Sigma_y$. An examination of equation (10) reveals that Σ is singular for all α for all $N > 1$. Therefore, Γ does not exist. For $\alpha_i = 1$ for some i, and $\alpha_j = 0$ for $i \neq j$, the usual one security equilibrium as in Kyle (1985) exists.■

Intuitively, when there is no noise trade, the order flows in different security markets are perfectly correlated, irrespective of the portfolio composition. Each security, therefore, reveals the same information. Coordination into one security market provides maximum thickness—and hence the lowest transaction costs.

While closed-form solutions are hard to find for the case of unequally informative securities, it is shown below that (almost) optimum profits can be earned off these traders by issuing the appropriate number of equally informative securities. Since the number of securities must be integer valued, the result provides only a level of profits close to the optimum. For any given number of securities, deviating from the equally informative strategy will lower per-unit profits slightly, as liquidity traders will meet their hedging needs in the markets that are cheaper to transact in. Therefore, with small perturbations with the number of securities provided in Result 5, the maximum can be achieved. Splitting assets to maximize profits off liquidity traders is profitable because it changes the ratio of noise to liquidity traders in each security market. As the number of markets increases, the mass of noise traders in each market thins out. This changes the depth of trading in the securities and affects the per-unit cost of liquidity traders. The following result characterizes the relationship between the parameters of the problem and the optimal number of securities to be issued.

Result 5 (Securities to Maximize the Profits Earned from Liquidity Traders)

Let $1 - F(r) = (r/M)$ for $0 \leq r \leq M$, and the volume of noise trade be $\gamma_z = 0$, given exogenously. Let N equally informative securities be designed to maximize profits solely off liquidity traders. Then

(i) In equilibrium, $\Sigma_{i,j} \Gamma_{ij}$ does not depend on ρ.

(ii) There exists an equilibrium with an optimal number N^* of identically distributed securities that, if issued, will maximize profits across all security designs. N^* is the integer closest to

$$N = \sqrt{\frac{\gamma_z \frac{M^2}{81}}{0.015\sigma_u^2 - \gamma_y \frac{4}{81}}}. \tag{16}$$

Proof

See the Appendix.

The intuition for statement (i) is as follows: changing ρ affects the correlation of the insider's information, and hence the correlation of his demands for the two securities. The market-maker responds to the change in observed total order flow by changing the cross-price sensitivities, Γ_{ij}. It is shown in the proof that, in equilibrium, increasing ρ lowers Γ_{ii} and raises Γ_{ij} by the *same* amount. Because liquidity traders hold a portfolio with weight $\alpha_i = 1$ in each security, their total costs of hedging per unit of endowment equal $0.25\ g_N = 0.25 \sum_{i=1}^{N} \sum_{j=1}^{N} \Gamma_{ij}$, which is unaffected by changes in ρ. This feature holds true for all continuous distribution functions, F(r).

Corollary 1

The optimal number of securities is decreasing in σ_u^2 and increasing in M, γ_y, and γ_z.

Proof

The result follows by taking partial derivatives of N with respect to the parameters.■

The somewhat surprising result is that N* is decreasing in σ_{υ}^2, the total information content of the underlying asset. This is because the security designer wants to ensure that per-unit transactions costs of the liquidity trader, g_N, equals M/3 in equilibrium, which is the level of transactions costs that maximizes profits. Increasing σ_{υ}^2 increases g_N for a given N. To lower this back to M/3, the designer lowers the number of securities. It is shown formally in the proof of Result 6 that g_N is increasing N. Intuition for this is provided following the statement of that result. The intuition behind the signs of the other parameters is straightforward. With the increase in the mass of each type of trader, the markets are thicker, resulting in a smaller Γ matrix and a smaller g_N. Therefore, the trader increases N to get g_N back to M/3, the optimal choice. With a larger M, the optimum g_N itself is higher; a higher M implies that there is a larger probability that hedging benefits exceed any given level of transactions costs, and the designer responds by setting a higher level of transactions costs. Therefore, in equilibrium, the designer increases the number of securities.

Securities also can be designed to maximize the consumer surplus of liquidity traders.

Result 6 (Securities to Maximize the Consumer Surplus of Liquidity Traders)

Let $1 - F(r) = \left(1 - \frac{r}{M}\right)$ for $0 \le r \le M$, and the volume of noise trade γ_z be given exogenously. Then it can be shown that the per-unit costs of liquidity traders are always increasing in the number of securities issued. Therefore, to maximize the consumer surplus of these traders, only the original unsplit security should be issued.

Proof

See the Appendix.

The intuition behind the result is as follows: noise trades thicken the market and provide a camouflage to both the market-maker and liquidity traders. Increasing the number of securities thins out the markets (since noise traders each use one securities market) and therefore increases the sensitivities of the market-maker's rule to order flow, correspondingly increasing the per-unit transactions costs of liquidity traders.

SALE TO INDIVIDUALS AND INSTITUTIONS

In the Sale to Individuals section, it is shown that, for a given number of securities N, profits earned from individual traders could be controlled by repackaging, and hence change the correlation coefficient between the receivables underlying the securities. In the Sale to Institutions section, it is shown that profits earned from institutional traders are immune to changes in the correlation coefficient and can be optimized only by changing the number of securities offered. The security designer, therefore, has an instrument of control to maximize profits from each type of trader. Therefore, a combination of strategies used in the previous two sections will control profits earned from each type.

Result 7 (Maximizing Profits of Noise and Liquidity Traders)

Let $1 - F(r) = \left(1 - \frac{r}{M}\right)$ for $0 \le r \le M$. Let securities be designed to maximize the sum of the profits made off the two groups of traders. Then, N* equally informative securities each mutually correlated with the others with correlation coefficient ρ* will maximize profits across all security designs. N* is the integer closest to

$$N = \sqrt{\frac{\gamma_z \frac{4}{9} \frac{M^2}{81}}{0.015\sigma_u^2 - 0.25\gamma_y \frac{4}{81}}}, \tag{17}$$

and ρ* is given by the unique solution to

$$\frac{(1-\rho)^{0.5}}{(1+(N^*-1)\rho)^{0.5}} = \frac{\gamma_z^{0.5}}{N^{*0.5}(N^*-1)} \left(\frac{2}{3} \frac{N^{*1.5}}{\sigma_u} M - \frac{1}{\left(\frac{\gamma_z}{N^*} + 0.25\, N^* \gamma_y \frac{4}{9} \right)^{0.5}} \right). \tag{18}$$

Proof

See the Appendix.

When securities are designed to minimize the consumer surplus of traders, it is again shown that issuing the single unsplit asset-backed security will be optimal.

Result 8 (Maximizing the Surplus of Both Noise and Liquidity Traders)

Let $1 - F(r) = \left(1 - \frac{r}{M}\right)$ for $0 \le r \le M$. Then the sum of the surplus of noise and liquidity traders will be maximized by issuing the single unsplit asset.

The result is somewhat surprising given Result 3, where costs of noise traders were lowered by increasing N. Intuitively, when both sorts of traders are present, each benefits from the thicker markets created by the other type. Increasing N lowers the volume of noise traders in each market, which, due to to a higher Γ_{ii}, leads to a decline in the volume of liquidity traders, which feeds back and lowers the mass of noise traders.

CONCLUSION

Asset securitization has grown rapidly in the last decade to include products backed by a range of receivables by varying agencies. Receivables are packaged, underwritten, and sold in the form of asset-backed securities, for many of which market-makers keep open markets. The securities are purchased by individuals and institutions to hedge various endowment risks. I provide a theory explaining how the information premium earned by the insider with information about the receivables can be controlled by the security designer. Depending on how uninformed agents allocate their resources across markets and the elasticity of their trading volume, optimal security design differs. It is found that, while profits earned off individuals can be optimized by changing the correlation coefficient between sets of receivables backing different securities, profits earned off institutions are immune to changes in the correlation but can be controlled by altering the number of securities created. A combination of the strategies enables the security designer to optimize profits or consumer surplus of each type of trader. Solutions for the optimal correlation between the receivables and the number of securities to be issued are provided as functions of (1) the objective of the security designer, which might be to maximize the profits earned off each kind of trader or to maximize the consumer surplus of hedgers; (2) the masses of each kind of trader in the market; and (3) the elasticities of traders' hedging volumes with respect to transactions costs.

APPENDIX

Mathematical Fact 1

Let the information matrix be given by

$$\Sigma_\upsilon = \begin{bmatrix} \sigma_i^2 & \rho\sigma_i^2 & \cdots & \rho\sigma_i^2 \\ \rho\sigma_i^2 & \sigma_i^2 & \cdots & \sigma_i^2 \\ \rho\sigma_i^2 & \rho\sigma_i^2 & \cdots & \rho\sigma_i^2 \\ \cdots & & & \\ \rho\sigma_i^2 & \rho\sigma_i^2 & \cdots & \sigma_i^2 \end{bmatrix}, \tag{19}$$

where σ_i^2 is constrained by equation (3) to satisfy σ_i^2 $(N + N\,(N-1)\,\rho) = \sigma_\upsilon^2$. This represents the information matrix Σ_υ, when N equally informative securities are issued. It can be verified that

$$\Sigma_{\upsilon ij}^{0.5} = \frac{\sigma_\upsilon}{N^{1.5}}\left[1 - \frac{(1-\rho)^{0.5}}{(1+(N-1)\rho)^{0.5}}\right], \qquad \forall i \neq j, \tag{20}$$

and

$$\Sigma_{\upsilon ii}^{0.5} = \frac{\sigma_\upsilon}{N^{1.5}}\left[1 + \frac{(N-1)(1-\rho)^{0.5}}{(1+(N-1)\rho)^{0.5}}\right]. \tag{21}$$

Proof of Lemma 1

Suppose the noise trader in the ith security market purchases z_{ji} units of the asset-backed security. The market-maker's rule implies that the expected price of the ith secuirty is $P_i = 1/N$, and in equilibrium the trader correctly anticipates the variance of the security, that is $\sigma_i^{e2} = \sigma_i^2$. The trader's return per unit of endowment equals

$$w_j = (1 - P_i\, z_{ji})\,(1 + \varepsilon_j) + z_{ji}\,(1 + \tilde{\upsilon}_i) + \mathbf{I}_{\{\tilde{r}_j - z_{ji}\,\Gamma_{ii} \geq 0\}}\,(\tilde{r}_j - z_{ji}\,\Gamma_{ii}).$$

Therefore, $E[w_j] = 1 + \mathbf{I}_{\{\tilde{r}_j - z_{ji}\,\Gamma_{ii} \geq 0\}}\,(\tilde{r}_j - \Gamma_{ii})$, and

$$\mathrm{Var}[w_j] = \left(1 - \frac{z_{ji}}{N}\right)^2 \sigma_\upsilon^2 + z_{ji}^2\,\sigma_i^2 - 2\left(1 - \frac{z_{ji}}{N}\right) z_{ji}\,\sigma_i\,\sigma_\upsilon.$$

Because the trader is infinitely risk averse with respect to consumption at t = 2, he or she plans to purchase the durable only if it is possible to reduce the variance of the return to zero. Since $\sigma_i^2 = K\,\sigma_\upsilon^2$, to reduce the variance of the return to zero when $\tilde{r}_j - \Gamma_{ii} \geq 0$, the trader must choose z_{ji} satisfying

$$1 - z_{ji}\left(\frac{2}{N} + 2\sqrt{K}\right) + z_{ji}^2\left(\frac{1}{N^2} + K + 2\frac{\sqrt{K}}{N}\right).$$

This quadratic equation has a unique solution, given in the statement of the lemma. Now integrating over the set of traders with independent risks implies that the aggregate uninformed trade is distributed normally with a mean of zero and a variance of $z_{ji}^2\ (1 - F(z_{ji}\,\Gamma_{ii})^2)$, which completes the proof.■

Proof of Result 2

Let the security designer issue N securities. Let each noise trader in security market i purchase z_{ji} units of the security he chooses. Profits of the insider when N securities are issued are given by

$$\pi_N^U = \sum_{i=1}^{N} \frac{\gamma_z}{N}\, z_{ji}\, \Gamma_{ii}\left(1 - \frac{z_{ji}\,\Gamma_{ii}}{M}\right)^2. \tag{22}$$

Suppose that $z_{ji}\,\Gamma_{ii}$ could be chosen by the security designer, unconstrained by any information criterion. Then the unique optimal (global) choice would be to choose $z_{ji}\,\Gamma_{ii}$ = M/3. Therefore, maximum unconstrained profits in the ith market are $\pi_i = \frac{4}{27} M \frac{\gamma_z}{N}$. Since there is an equal mass $\frac{\gamma_z}{N}$ in each security market, maximum unconstrained profits with N securities are $\pi_N^U = \frac{4}{27} M$, clearly independent of N. This proves statement (i).

For N = 1, using equation (14), the market depth parameter λ satisfies $\lambda\left(1 - \frac{\lambda}{M}\right) = \frac{\sigma_\upsilon}{\gamma_z^{0.5}}$. The solutions of this equation are

$$\hat{\lambda} = \frac{M}{2}\left(1 \pm \sqrt{1 - 4\frac{\sigma_\upsilon}{\gamma^{0.5}}\frac{1}{M}}\right) \tag{23}$$

that are not generically equal to M/3, the unique unconstrained optimum. This proves statement (ii).

I now show that, with $N \geq 2$ equally informative securities, the two-security unconstrained maximum can be attained. By Result 1, $\Gamma = 0.5\, \Sigma^{-0.25}\, \Sigma_\upsilon^{0.5}\, \Sigma^{-0.25}$. The noise matrix Σ is given by $\mathrm{Diag}\left(\frac{\gamma_z}{N} z_{ji}^2 \left(1 - \frac{z_{ji}\Gamma_{NN}}{M}\right)^2, \ldots, \frac{\gamma_z}{N} z_{ji}^2 \left(1 - \frac{z_{ji}\Gamma_{11}}{M}\right)^2\right)$, where, by Lemma 1, $z_{ji} = \frac{1}{\frac{1}{N} + \sqrt{K}}$ for $i = 1, \ldots, N$. The elements of the matrix Σ_υ are given by equations (20) and (21). Result 1 therefore implies that, in equilibrium,

$$z_{ji}\, \Gamma_{ii} \left(1 - \frac{z_{ji}\, \Gamma_{ii}}{M}\right) = 0.5 \frac{1}{N} \frac{\sigma_\upsilon}{\gamma_z^{0.5}} \left[1 + \frac{(N-1)(1-\rho)^{0.5}}{(1+(N-1)\rho)^{0.5}}\right] \tag{24}$$

for $i = 1, \ldots, N$. Now, setting $z_{ji}\, \Gamma_{ii} = M/2$ into equation (24) implies that, if the security designer issues N equally informative securities with correlation ρ between them satisfying

$$\frac{4}{9} M = 0.5 \frac{1}{N} \frac{\sigma_\upsilon}{\gamma_z^{0.5}} \left[1 + \frac{(N-1)(1-\rho)^{0.5}}{(1+(N-1)\rho)^{0.5}}\right],$$

then, in equilibrium, the unconstrained maximum with N securities will be attained. Now simplifying completes the proof of statement (iii).■

Proof of Result 3

The demands given in Lemma 1 completely hedge the risk of each trader at a cost of $z_{ji}\, \Gamma_{ii}$. To maximize the total consumer surplus of these traders, we need to find the security design that reduces the hedging cost to zero. In equilibrium, equation (24) is satisfied. If $\rho = 0$, then the right-hand side of equation (24) equals $\frac{\sigma_\upsilon}{\gamma_z^{0.5}}$, which is independent of N. Therefore, the roots of the equation, which determine $z_{ji}\, \Gamma_{ii}$ in equilibrium, do not depend on N; consequently, the transactions costs incurred by traders and their consumer surplus is invariant to N. If the designer creates N equally informative and perfectly positively correlated securities, then $z_{ji}\, \Gamma_{ii} \left(1 - \frac{z_{ji}\, \Gamma_{ii}}{M}\right) = 0.5 \frac{\sigma_\upsilon}{\gamma_z^{0.5}} \frac{1}{N}$. Now, letting N tend to infinity implies that there are two equilibria; in the first the hedging cost tends to zero and in the second to M. In the second equilibrium, volume tends to zero; therefore, consumer surplus is minimized. Letting the security designer choose the equilibrium completes the proof of statement (ii).■

Proof of Result 5

The first part of the proof shows that the costs of liquidity traders do not depend on the correlation in the information content of different securities, as long as these securities are equally informative. Let the information matrix be given by equation (19). By Result 1, $\Gamma_N = 0.5\, \Sigma_N^{-0.25}\, \Sigma_\upsilon^{0.5}\, \Sigma_N^{-0.25}$. Using the expression for $\Sigma_\upsilon^{0.5}$ from Mathematical Fact 1 and multiplying through implies that Γ_N is of the form

$$\Gamma_N = 0.5 \frac{\sigma_\upsilon}{N^{1.5}} \Sigma_N^{-0.25}. \tag{25}$$

$$\left(E_N + \begin{bmatrix} (N-1)\, g(\rho) & -g(\rho) & \cdots & -g(\rho) \\ -g(\rho) & (N-1)\, g(\rho) & \cdots & -g(\rho) \\ -g(\rho) & -g(\rho) & \cdots & -g(\rho) \\ \cdots & & & \\ -g(\rho) & -g(\rho) & \cdots & (N-1)\, g(\rho) \end{bmatrix}\right) \Sigma_N^{-0.25},$$

where $g(\rho) = \dfrac{(1-\rho)^{0.5}}{(1+(N-1)\,\rho)^{0.5}}$, and E_N is an $N \times N$ matrix of ones. To simplify notation, let $\sum_{i=1}^{N} \sum_{i=1}^{N} \Gamma_{Nij} = g_N$. Let us conjecture that each liquidity trader buys a unit of each security per unit of endowment security, that is, $\alpha_i = 0.5$ for $i = 1, \ldots, N$. We shall verify this conjecture below. Under this assumption, $C_y(\Gamma) = 0.25\, g_N$, and $\Sigma_y = 0.25\, E_N\, \gamma_y\, (1 - \frac{1}{M} g_N)^2$, and $\Sigma_z = I_N \frac{\gamma_z}{N}$. Therefore,

$$\Sigma_{Nii}^{-0.25} = \frac{N-1}{N} \frac{1}{\left(\frac{\gamma_z}{N}\right)^{0.25}} + \frac{1}{N\left(\frac{\gamma_z}{N} + 0.25\, N\, \gamma_y \left(1 - 0.25 \frac{1}{M} g_N\right)^2\right)^{0.25}} \qquad \forall i,$$

$$\Sigma_{Nij}^{-0.25} = -\frac{1}{N} \frac{1}{\left(\frac{\gamma_z}{N}\right)^{0.25}} + \frac{1}{N\left(\frac{\gamma_z}{N} + 0.25\, N\, \gamma_y \left(1 - 0.25 \frac{1}{M} g_N\right)^2\right)^{0.25}} \qquad \forall i \neq j.$$

Both terms do not depend explicitly on ρ, but only implicitly through g_N. Now substituting for Σ_{Nii} and Σ_{Nij} into equation (25) implies that

$$\frac{N^{1.5}}{0.5\sigma_{\upsilon}}\Gamma_{ii} = (N-1)\,g(\rho)\frac{1}{\left(\frac{\gamma_z}{N}\right)^{0.5}} + \frac{1}{\left(\frac{\gamma_z}{N} + 0.25\,N\,\gamma_y\left(1-0.25\frac{1}{M}g_N\right)^2\right)^{0.5}} \qquad (26)$$

$$\frac{N^{1.5}}{0.5\sigma_{\upsilon}}\Gamma_{ij} = -\,g(\rho)\frac{1}{\left(\frac{\gamma_z}{N}\right)^{0.5}} + \frac{1}{\left(\frac{\gamma_z}{N} + 0.25\,N\,\gamma_y\left(1-0.25\frac{1}{M}g_N\right)^2\right)^{0.5}} \quad \forall i \neq j. \qquad (27)$$

Because $\Gamma_{ii} = \Gamma_{jj}$ $\forall i,j$ and $\Gamma_{ij} = \Gamma_{kl}$ $\forall i \neq j$ and $k \neq l$, and therefore the equal portfolio weight in each asset is optimal. Now summing the elements of equations (26) and (27) implies that $C_y(\Gamma_N) = 0.25\,g_N$ is the solution of

$$0.25\,g_N = 0.125\frac{\sigma_{\upsilon}}{N^{1.5}}\frac{1}{\left(\gamma_z + 0.25\,N^2\,\gamma_y\left(1-0.25\frac{1}{M}g_N\right)^2\right)^{0.5}}, \qquad (28)$$

which is independent of ρ (terms containing ρ exactly cancel out). This proves statement (i).

Equation (28) provides the "information constraint" for the security designer on per-unit transaction costs. Simplifying equation (28) implies that

$$g_N^2 = \frac{0.016\,\sigma_{\upsilon}^2\,N^2}{\left(\gamma_z + 0.25\,N^2\,\gamma_y\left(1-0.25\frac{1}{M}g_N\right)^2\right)}. \qquad (29)$$

Suppose the security designer chooses the per-unit transaction costs $0.25\,\hat{g}$ of liquidity traders to maximize profits off these traders. Given their Σ_y matrix, profits would be $\pi_L = 0.25\,\hat{g}\,(1-0.25\frac{1}{M}\hat{g})^2\,\gamma_y$. Given the objective, profits will be maximized by choosing $0.25\,\hat{g} = M/3$ in equation (29). Substituting $0.25\,g_N = M/3$ into equation (29) then provides the expression in the statement of the result.■

Proof of Result 6

With N equally informative securities, irrespective of the correlation between them, the costs of liquidity traders are given by equation (29). Simplifying further, we can write $\Omega(g_N^2, N) = 0$, where

$$\Omega(g_N^2, N) = g_N^2 \gamma_z + 0.25\, N^2 \gamma_y g_N^2 (1 - 0.25\ \frac{g_N}{M})^2 - 0.25\ \sigma_u^2 N^2.$$

Now, by the implicit function theorem,

$$\frac{\partial g_N}{\partial N} = -\frac{\frac{\partial \Omega}{\partial N}}{\frac{\partial \Omega}{\partial g_N}} = \frac{2N\left[0.25\,\gamma_y\, g_N^2 \left(1-0.25\,\frac{g_N}{M}\right)^2 - 0.25\sigma_u^2\right]}{2g_N\gamma_z + 2\; 0.25\, N^2\, g_N\, \gamma_y \left(1-0.25\,\frac{g_N}{M}\right)\left(2-0.25\,\frac{g_N}{M}\right)}.$$

Because $g_N \geq 0$, and $\left(1-\frac{g_N}{M}\right) \geq 0$ (since it is a probability), the denominator is positive; therefore, $\text{sign}\left(\frac{\partial g_N}{\partial N}\right) = -\text{sign}\left(\gamma_y\, g_N^2 \left(1-\frac{g_N}{M}\right)^2 - 0.25\sigma_u^2\right)$. However, because equation (29) holds, this equals $\text{sign}\left(-\frac{g_N^2\, \gamma_z}{N^2}\right)$, which is clearly negative. Therefore, we have shown that g_N is always increasing in N, and to minimize the transactions costs of liquidity traders, it is optimal to issue the smallest N possible; that is, N = 1. Since the lowest transactions costs also induce the highest benefit from hedging, this maximizes consumer surplus.■

Proof of Result 7

The proof dichotomizes into parts similar to those of Results 2 and 5. By Result 1, we can write the equilibrium conditions similar to equations (26) and (27). To maximize profits off each kind of trader, the security designer must adjust ρ and N to ensure that $z_{ji}\,\Gamma_{ii} = M/3$ and $0.25 \sum_{i=1}^{N} \sum_{j=1}^{N} \Gamma_{ij} = M/3$. Equilibrium conditions can now be obtained by using these conditions in equations (26) and (27), replacing γ_z with $\gamma_z\, 4/9$. Summing the depths implies that profits off liquidity traders again are independent of ρ. Completely analogously to Result 5, the optimal number of securities N* is given by equation (17) in the statement of the result. Similarly, substituting these optimal quantities in equation (26) and using N* above provides the expression for ρ in equation (18) in the statement of the result.■

Proof of Result 8

Equation (26) implies that Γ_{ii} is lowest when each security has the lowest information content—that is, $\rho = 1$—in this case, $g(\rho)$ equals zero and the first term on the right-hand side disappears. Comparing with equation (27) implies that $\Gamma_{ii} = \Gamma_{ij}$.

Therefore, $g_N = N^2 \Gamma_{ii}$. Substituting $\left(1-\frac{z_{ji}}{M}\right)^2 \gamma_z$ into equation (29) in place of γ_z and taking the implicit derivative as in the proof of Result 6 again implies that g_N is increasing in N. Therefore, Γ_{ii} is increasing in N, and so is $z_{ji}\,\Gamma_{ii}$, because z_{ji} is increasing in N. Therefore, each trader's costs are increasing in N, and thus lowest costs will be attained when the single asset is issued.■

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