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Pricing the strategic value of puttable securities in liquidity crises[☆]

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Abstract

Puttable security holders have a de facto first claim on the firm's liquid assets and can threaten to force solvent issuers to bear financial distress costs. Their threatening power implies that the puts have a *strategic* value larger than their *intrinsic* value. Strategic value depends on the issuer's size, potential distress costs, and the distribution of put ownership relative to the firm's liquidity position. The analysis of Kmart's put-induced crisis in 1995, and a calibration to observed secondary market yield reductions on poison put bonds, shows that strategic value is an important determinant of payouts received by bondholders. © 2001 Elsevier Science S.A. All rights reserved.

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1. Introduction

Puttable securities have been issued under various forms by corporations in recent years, often motivated by the need to protect investors against major declines in value, and thus they potentially thicken markets by drawing in more conservative classes of investors. However, as revealed in several cases in the financial media in recent years, such securities can often exacerbate liquidity crises for solvent issuers because they provide security holders a de facto first claim on the firm's available funds following a major credit-debilitating event, giving them the ability to force the issuer to either incur costly asset sales, raise its borrowing costs, or, in the most acute cases, file for a costly bankruptcy. Holding a threatening weapon can induce gaming behavior among such security holders and often these bondholders can use their strategic position to extract payments from equity holders in excess of the contracted payment at the time of exercise. Along similar lines, Dunn and Spatt (1999) suggest that the ability of putholders to extract a side payment from the mortgage borrower at the time of a costly refinancing partly explains why such lending contracts are typically callable but not puttable. In this paper, I model multilateral negotiations between puttable bondholders and the firm in an explicit financial distress setting and show that the strategic value of a typical bondholder depends on his size, on the issuer's potential financial distress costs, and on the distribution of ownership of puttable debt relative to the firm's liquidity position. The analysis of Kmart's put-induced crisis in 1995, and a calibration to observed secondary market yield reductions on poison put bonds, shows that the strategic value is an important determinant of payouts received by puttable bondholders.

Two highly publicized negotiations illustrate the severity of the possibilities discussed. In September 1995, Kmart Corp. was on the brink of a bankruptcy filing because of the downgrading of its debt to nearly speculative grade by the major rating agencies. A further downgrade would trigger the put of \$550 million of poison put bonds. Despite having more than \$1 billion in cash and other marketable securities, and an overall asset value of \$16 billion, Kmart was prohibited by covenants written by senior bank lenders to accelerate payments on more than one-fifth of the outstanding puttable debt. The covenant limited the extent of the effective violation of seniority through put exercise on junior debt, thus preserving the value of senior holders amid declines in firm value. Therefore, exercise of more than a critical proportion would force Kmart into

a bankruptcy filing, even though it was otherwise solvent. Kmart reached a settlement with the lenders, offering bondholders a lump sum to surrender their options. It was reported in the press that a total of \$98 million was paid to puttable bondholders, and that an insurance company was offered five times as much per dollar as banks holding similar bonds. The settlement was accepted by bondholders and Kmart's shares soared immediately (see, among other media articles at the time, *Bloomberg Financial Service*, Nov. 15, 1995, pp. 1–3).

More recently, in August 1999, General American Life Insurance Corp. (Gen. Amer.), a company with \$29 billion in assets, sought the protection of Missouri insurance regulators because it was unable to meet the liquidity needs of a put of \$5 billion of its funding agreements (FAs) held by several large money market mutual funds. The FAs were not publicly traded, but were puttable at par value, and investors decided on a simultaneous put following a rating downgrade from investment to speculative grade by Moody's. Under the terms of the put contract, Gen. Amer. was obliged to pay back the funds within seven days of the put, but it soon became evident that it would be unable to meet all withdrawals over this short period because of the illiquidity of its assets. While a few of Gen. Amer.'s customers withdrew their policies and market participants fretted over the possible default on money funds (a hitherto unknown possibility), the largest mutual funds obtained letters of credit from guarantors for a fee, thus keeping the price of these funds near par value, and agreed to a month-long grace period with Gen. Amer. in return for an immediate payment of about 10% of face value ("There's something askew when people start losing sleep over, of all things, money-market funds" writes S. Ward in an August 21 1999 article in *Dow Jones News Service via Dow Vision*). Because the money funds holding these FAs were prohibited by regulation to strip the put options at a strategic price and hold unprotected loans (see, e.g., SEC Release 34-359991), Gen. Amer. had to bear the costly asset sales to meet its liquidity needs, and several days later was bought out by Metropolitan Life Insurance, which assumed the FAs as part of the deal.

Using recent results in multilateral bargaining theory (Hart and Mas-Colell, 1996), I solve for the strategic values of put options at the date of exercise. The solution of the alternating-offers non cooperative game coincides with a cooperative game theory concept, the Shapley value, which assigns a strategic value to each player equal to his expected marginal value. The expectation is with respect to the uniform probability distribution over all coalitions that can potentially be formed. Under this distribution, the 'probability' of each bondholder being in a threatening coalition is shown to depend on the distribution of puttable debt held by various bondholders and the position of the liquidity trigger relative to the amounts of debt held by different bondholders. When the debt is held by a continuum of infinitesimal bondholders, the probability of a given bondholder being in a threatening coalition equals the ratio of liquidity to puttable debt, a convenient back-of-the-envelope calculation for publicly held bonds. The intrinsic and strategic values do not coincide for a ratio of liquidity

to puttable debt less than one. In general, it will be seen that the strategic values of players depend on the following variables at the time of negotiations: (1) the size of each lender's holding of the company's debt, (2) the size of the 'effectively' liquid assets of the company relative to the amount of puttable debt outstanding, (3) the expected costs of bankruptcy, and (4) the value of nonputtable bonds and equity. The valuation method can be extended to price similar contracts. As a demonstration, I explicitly formulate the strategic values of reset bondholders who can impose a cost on equity holders because the firm commits to raising the coupon on the bonds at a fixed date or following a major event.

The structural form bond-pricing framework of Leland and Toft (1996) is used to value put bonds prior to the exercise date. The major innovation in my framework is to formulate the strategic value of all players at the time of exercise, and to use these values as boundary conditions in the *ex ante* valuation. A calibration exercise reveals that the strategic valuation of poison puts was an important determinant in explaining the secondary market yield reductions that were observed by Crabbe (1991) on investment-grade poison put bonds issued in the late 1980s.

The phenomenon of size- and distribution-dependent strategic valuation of securities at the time of repurchase by the firm is first discussed in the context of a sinking fund for bonds by Dunn and Spatt (1984). These authors establish the threatening power of large bondholders who become 'pivotal' players (the repurchase of whose bonds is essential to the requirement of the sinking fund) by cornering a substantial share of actively traded bonds and being able to extract more than the market price of their bonds from the firm. Despite different economic circumstances leading up to the crises, it turns out that the strategic positions of sinking fund and puttable bondholders are quite similar. In particular, if the firm's available liquid resources are smaller than the amount of puttable debt outstanding, then even if all the puttable debt is held by infinitesimal bondholders, the strategic value of their options exceeds their intrinsic value. Similarly, if the sinking fund requirement exceeds the active supply of bonds owned by infinitesimal bondholders, each bondholder can demand more than the market price of the bonds (Dunn and Spatt, 1984, Proposition 1). Moreover, if small bondholders cannot threaten the company on their own, while a large bondholder can, then only the large bondholder has an incremental strategic value. Similarly, in the sinking fund market smaller bondholders can be inessential if the debt held by larger players exceeds the sinking fund requirement (Dunn and Spatt, 1984, Section 2).

In a broader context, the paper adds to a growing literature on strategic issues in the pricing of risky debt. Franks and Torous (1989) observe that the violation of absolute priority in debt contracts is an indicator of strategic behavior among various creditors and equity holders of the firm in and before bankruptcy. Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) provide conditions under which the firm offers a *strategic debt service*, which in

many situations is smaller than the promised sum to its debtholders. Because I use an alternating-offers framework in which parties make repeated counter-offers to each other until settlement, the effect of the first-mover advantage assigned to some player(s) in some of the earlier papers is smaller, and completely disappears in the limiting case of instantaneous rounds of bargaining or zero probability of negotiation breakdown. The use of the alternating framework does not imply that the entire strategic value of the first mover disappears, but rather that it can be distributed among other players. Hence the qualitative results of the other papers remain valid in this framework.

The plan for the paper is as follows. General characteristics of the puttable securities market are described in Section 2. Section 3 describes the capital structure of the firm with puttable bonds outstanding, and the frictionless structure is adjusted to incorporate the timing and rules of the bargaining game that arises when put-triggering covenants are violated. Section 4 defines intrinsic and strategic values for put options. Strategic values are computed when the puttable debt is divided equally among an arbitrary number of bondholders (Section 4.1), and when one large bondholder holds a significant amount of the debt (Section 4.2). The model used to study the strategic value of reset bondholders and some qualitative implications for RJR Nabisco's reset in 1990 are also presented (Section 4.3). The *ex ante* pricing of poison puts prior to the crisis and a model calibration to observed yield reductions on poison put bonds are in Section 5. Strategic values of poison put bondholders are calibrated to Kmart's situation in Section 6. I conclude, summarize the implications of the analysis, and suggest some possible extensions in Section 7.

2. General characteristics of the puttable securities market

Puttable securities have been issued under several forms. Poison put bonds were initially designed to make firms less attractive as takeover targets and to thus provide an additional mechanism for strengthening managerial resistance to hostile bids (see, e.g., Cook and Easterwood, 1994). Their popularity increased rapidly in the late 1980s after leveraged buyouts lowered the credit quality of many firms. Poison put bonds have remained popular: data from the *Securities Data Company* reveal that about \$141 billion of poison puts were issued in the period 1991–1997, accounting for almost 15% of all nonfinancial bond issuance in the sample. The median firm issues a face value of about \$250 million, potentially placing itself in situations in which it could face an outflow of liquid resources of this magnitude. At first, most puts were exercisable upon the threat of a hostile takeover, but it became common to make the bonds exercisable upon any change in ownership or recapitalization and a 'large' downgrading of the debt by either Standard and Poor's or Moody's (for the legal complexities characterizing hostile deals, see *The Wall Street Journal*, November 16, 1986,

p. C1; *Forbes Magazine*, issue 22, 1988, p. 117; Nash et al., 1997). In the 1990s, poison put bonds (such as that of Kmart Corp.) that were exercisable on a rating change alone evolved, thus protecting the bondholders from declines in credit quality irrespective of the cause.

FAs (funding agreements) are fixed and floating rate investment contracts issued as general account or guaranteed separate account obligations by insurance companies to the non qualified institutional market. FAs have become a popular short-term financing vehicle among insurance companies in recent years and most have embedded protective puts that help attract considerable assets from money market funds, and other institutional lenders. For more background information on FAs, see *Moody's Investor Service Global Credit Research*, 1998 report on funding agreements. The actual size of the FA market is difficult to determine in that insurance companies account for them in an inconsistent manner. Moody's *Stable Value* survey conducted in 1998 reveals that roughly \$22 billion were issued in 1996 and 1997 and the amount then outstanding was about \$40 billion, though Moody's and other industry experts have placed the figure closer to \$60 billion. The FA product in the aggregate constitutes a very small part of money market funds, which are in excess of \$1 trillion. However, as seen in the case of Gen. Amer., a liquidity crisis can trigger the failure of a company with assets that are considerably larger in magnitude.

3. The model

3.1. A structural form model for bond pricing

Assumption 1 (Asset value process). The firm has productive assets whose unleveraged market value, V , under the risk-neutral measure follows a continuous diffusion process:

$$\frac{dV}{V} = [r - \delta] dt + \sigma dz, \quad (1)$$

where r is a constant short-term interest rate, δ is the constant fraction of value paid out to security holders, and z is a Weiner process. Dividends paid to equity holders equal the total payout, δV , less the coupons to all debt securities.

Assumption 2 (Capital structure of the firm). The firm has nonputable and putable (under conditions to be defined) junior coupon debt with face values D^{NP} and D and senior debt with face value B outstanding, respectively. All debt claims mature at date T . At T , the firm will issue new senior debt of face

value B and new junior debt of $D + D^{\text{NP}}$, and the same issuance pattern is repeated at future maturity dates. A claim of seniority S pays a continuous coupon yield of c_S .

Assumption 3 (Reorganization boundary). Following standard structural form models (see, e.g., Leland 1994, Longstaff and Schwartz, 1995), I assume that bondholders of seniority S , have a minimum net worth protection covenant of $0 \leq \alpha_S \leq 1$ per dollar of face value of debt. In addition, equity holders can extract a fraction α_E of the value of the assets net of any bankruptcy costs. Bankruptcy costs are a fraction Φ of the value of the firm. The covenant implies that the debts of the firm are only permitted to be ‘rolled over’ by all liability holders if the value of the firm exceeds V_D , given by

$$V_D = \frac{1}{(1 - \Phi)(1 - \alpha_E)} [\alpha_B B + \alpha_D (D + D^{\text{NP}})], \quad (2)$$

which is the smallest asset value that ensures the minimum payout to each class of liabilities. When the asset value hits V_D , the firm files for bankruptcy immediately.

3.2. The pricing of contingent claims

I use the framework of Leland and Toft (1996) to price the nonputtable coupon debt outstanding and find the discounted value of expected bankruptcy costs under the risk-neutral measure in a standard continuous-time setting.¹ Using the results in Leland and Toft, the price of a dollar of junior debt at time t that matures at T when the value of the firm is V is given by

$$P_D(V, V_D; t, T) = \frac{c_D}{r} + e^{-r(T-t)} \left(1 - \frac{c_D}{r} \right) (1 - F(V, V_D; t, T)) + \left[\alpha_S - \frac{c_D}{r} \right] G(V, V_D; t, T). \quad (3)$$

$G(V, V_D; t, T) = \int_{s=t}^T e^{-r(s-t)} f(V, V_D; t, s) ds$; $f(V, V_D; t, s)$ is the density of the first passage time s to V_D from V at time t of the process in Eq. (1); and

¹ Since the seminal work of Black and Cox (1976) and Merton (1974) in pricing risky debt under the structural approach, several other papers with similar structures have been written. I do not attempt an exhaustive survey of the pricing literature, but notably the following papers have generalized the pricing problems in different directions: Brennan and Schwartz (1984), Briys and de Varenne (1997), Brennan et al. (1989), Leland (1994), Leland and Toft (1996), and Longstaff and Schwartz (1995).

$F(V, V_D; t, T) = \int_{s=t}^T f(V, V_D; t, s) ds$ is the cumulative distribution function of the first passage time T to V_D from V at time t .²

The discounted expected value of future bankruptcy costs under the risk-neutral measure can be calculated as that of a risky bond that does not pay any coupon and provides a return of ΦV_D when V hits V_D ; hence,

$$\mathcal{B}(V, V_D; t, T) = \int_t^\infty e^{-r(s-t)} \Phi V_D f(V, V_D; t, s) ds = \Phi V_D G(V, V_D; t, \infty). \quad (7)$$

In the remainder of the paper, I will simplify notation by referring to the pricing and bankruptcy functions without their arguments; for example, $P_D(V, V_D; t, T)$ is referred to simply as P_D . The asset value V and the time t will be evident from the situation being discussed.

3.3. The put-induced crisis

Assumption 4 (Put exercise boundary). The junior puttable debt can be put back to the firm at par (\$1 for face value of debt of \$1) at the *first time* the firm's long-term credit rating hits a prespecified rating category R . For most poison puts written, the holder of the put has a right to put the bond on the day of and

²The density of default, $f(\cdot, \cdot)$, is given by

$$f(V, V_D; t, T) = \frac{b}{\sigma\sqrt{2\pi}(T-t)^3} \exp\left[-\frac{1}{2}\left(\frac{b + (r - \delta - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}\right)^2\right], \quad (4)$$

$F(\cdot, \cdot)$ and $G(\cdot, \cdot)$ are given by

$$F(V, V_D; t, T) = N[h_1(t, T)] + \left(\frac{V}{V_D}\right)^{-2a} N[h_2(t, T)] \quad (5)$$

and

$$G(V, V_D; t, T) = \left(\frac{V}{V_D}\right)^{-a+z} N[q_1(t, T)] + \left(\frac{V}{V_D}\right)^{-a-z} N[q_2(t, T)], \quad (6)$$

in which

$$q_1(t, T) = \frac{(-b - z\sigma^2(T-t))}{\sigma\sqrt{T-t}}, \quad q_2(t, T) = \frac{(-b + z\sigma^2(T-t))}{\sigma\sqrt{T-t}},$$

$$h_1(t, T) = \frac{(-b - a\sigma^2(T-t))}{\sigma\sqrt{T-t}}, \quad h_2(t, T) = \frac{(-b + a\sigma^2(T-t))}{\sigma\sqrt{T-t}},$$

$$a = \frac{r - \delta - \sigma^2/2}{\sigma^2}, \quad b = \ln\left(\frac{V}{V_D}\right), \quad z = \frac{[(a\sigma^2)^2 + 2r\sigma^2]^{\frac{1}{2}}}{\sigma^2},$$

and $N(\cdot)$ is the cumulative density function of a standard normal random variable.

for a few days after the triggering event/rating change; see, for example, clause (iv) on p. 66 of the prospectus for Americold's 12.875% senior subordinated notes due 2008. Let V_R be the asset value satisfying the property $F(V_R, V_D; t, t + T_L) = \pi^R(T_L)$; $\pi^R(T_L)$ is the default probability associated with Moody's credit rating R with T_L periods to maturity, and T_L can be chosen arbitrarily.³

Let τ_P be the hitting time of V to V_R ; τ_P is the time at which the firm's credit rating hits the critical rating at which put options first become exercisable. Suppose the *effectively liquid* assets of the firm at τ_P are smaller than the amount of debt to be paid back to the puttable bondholders. The amount of cash and securities on the balance sheet might not correspond to the effective liquidity of the company; often, covenants written on the firm's bank debt restrict the use of this cash in distress situations. The equity holders of the firm are assumed to be represented collectively by the management and shall be referred to as the firm. The firm enters into a bargaining process with the puttable bondholders in an attempt to buy back the put options and hence delay bankruptcy. It is assumed that all the puttable bondholders are fully cognizant of the balance sheet of the company, and therefore the bargaining game that follows is under complete information.

Assumption 5 (Payoffs in bankruptcy at rating boundary). If the firm accelerates the payments on more than a trigger amount, \mathcal{T} , of outstanding puttable bonds, then it is forced into a bankruptcy filing. Payments for senior debt, B , junior nonputtable debt, D^{NP} , junior puttable debt, D , and equity, E , in this case are

$$B_F = \min\{(1 - \Phi)V_R, B\}, \quad (8)$$

$$D_F^{\text{NP}} = \frac{D^{\text{NP}}}{D^{\text{NP}} + D} \min\{(1 - \Phi)V_R - B_F, D^{\text{NP}} + D\}, \quad (9)$$

$$D_F = \frac{D}{D^{\text{NP}} + D} \min\{(1 - \Phi)V_R - B_F, D^{\text{NP}} + D\}, \quad (10)$$

and

$$E_F = (1 - \Phi)V_R - B_F - D_F^{\text{NP}} - D_F. \quad (11)$$

³Note that the rating does not depend on the proportion of junior debt that is puttable if a condition to be introduced holds at the rating boundary. In the proof of Result 1 below, I will show that when the rating boundary is sufficiently high that the firm's asset value net of potential bankruptcy costs exceeds the face value of all outstanding debt (Condition 1), then successful negotiations are possible. At the time of exercise, there is a lump sum payment from equity holders to bondholders, but no immediate bankruptcy. If the firm writes put options at a rating boundary where Condition 1 does not hold, then it is possible that negotiations break down, and then the ratings will have to be recalibrated for default at V_R .

Because the bonds are put when the value of assets is at the rating boundary, which is higher than the default boundary, payments are made in order of absolute priority.

Assumption 6 (Bailout loan). If $P_D(V_{\tau_p}, V_D; \tau_p, T) < 1$, and D_P of the putable debt is put at τ_p , and the firm buys out the put options on the remainder, then $D_C = D - D_P$ for an aggregate payment of X_C . Therefore, the firm's immediate financing needs equal $\mathcal{L} = D_P + X_C$. It is assumed that the firm will be able to raise *unprotected* debt ($\alpha_L = 0$) at par value to cover these immediate needs as long as the value of the equity after the loan remains positive; that is,

$$V_{\tau_p} - \mathcal{B} - B - (D^{\text{NP}} + D_C)P_D - \mathcal{L} > 0. \quad (12)$$

The firm will rationally not choose a larger loan than that implied by the inequality because it has the choice of immediately defaulting and getting the payoff $E_F \geq 0$ in Eq. (11). Issuance at par implies that the coupon payment, c_L , is set to satisfy $P_L(V, V_D; \tau_p, T) = 1$ using Eq. (3). Because the lack of protection on this loan means that its issuance will not increase the default barrier, I call it a bailout loan.

I assume that the immediate financing needs are met by issuing a bailout loan. Instead, I could assume that the firm issues new equity. If there are no distress costs of issuing equity, the firm is indifferent between raising debt and equity.

Lemma 1. *If the bailout loan is made at par, and equity can be issued at no distress costs, then equity holders are indifferent to the method of financing.*

Proof. Let the value of the equity holders with the bailout loan priced at par be $v_L = V_{\tau_p} - \mathcal{B} - B - (D^{\text{NP}} + D_C)P_D - \mathcal{L}$. Similarly, let $v_E = V - \mathcal{B} - B - (D^{\text{NP}} + D_C)P_D$ be the value of the equity holders under equity financing. If equity can be issued under no distress costs, then a fraction ε of the ownership is sold to new equity holders satisfying $\varepsilon v_E = \mathcal{L}$. Then, the value to the ex ante shareholders equals $(1 - \varepsilon)v_E$, which is easily seen to equal v_L .

Intuitively, because the bailout loan is senior to equity, the residual claim of the current equity holders will be the same for each of the two methods. Since it is unlikely that the distress costs of issuing equity at the time of a financial crisis are non zero, I shall assume that the financing need is met through a bailout loan.

To simplify the analysis of pricing the put options prior to the crisis, I need four additional assumptions:

Assumption 7 (Subsequent financing). After the resolution of the crisis, the firm will issue new junior debt of D_P with protection α_p to restore the total amount of

junior debt outstanding to pre-crisis levels. The same amount of the bailout loan will be paid down. Therefore, irrespective of the amount of outstanding putable debt that is actually put at the time of the crisis, the default boundary will remain as in Eq. (2).

Assumption 8 (Unchanging distribution of putable bonds). The distribution of the ownership of putable debt remains unchanged until the time to maturity. When the bonds are publicly held by infinitesimal bondholders, the assumption is consistent with trading that leaves all bondholders small.

Assumption 9 (Floating-rate senior debt). Senior debt has a continuously floating coupon, c_D , to ensure $P_B = 1$ under the pricing equation (3). The valuation of reset bonds with discrete adjustments to coupons, which cause lumpy increases in the issuer's financing costs, are studied in Section 4.3.

Assumption 10 (Preferences). All players are risk-neutral. This assumption enables me to specify the value of a coalition of players without concerns about the "strategic risk" that arises in alternating-offer bargaining games (see, e.g., Roth, 1985), thus considerably simplifying the bargaining solution. In the game to be defined in Section 4, the proposer faces the risk of being dropped for the remainder of the game if his offer is not accepted by each of the other players; a multilateral bargaining solution with risk-averse players is beyond the scope of this paper. The assumption also implies that the asset value process is as specified in (1) under agents' objective measure.

I have yet to describe the formal rules of the bargaining game. However, looking ahead, I introduce a condition under which successful negotiations are possible.

Condition 1. The rating boundary, V_R , is large enough that the putable bondholders make full recovery of face value if the firm is forced into immediate bankruptcy by the exercise of the puts: $D_F = D$. The condition is equivalent to $(1 - \Phi)V_R \geq B + D^{\text{NP}} + D$.

One can easily generate sets of parameter values such that, for a given capital structure of debt, Condition 1 will be satisfied. An example follows.

Example 1 (A simple restriction on parameters for Condition 1). The rating boundary V_R must satisfy $F(V_R, V_D; 0, T_L) = \pi^M(T_L)$. Keeping all other parameters constant, there exists a lower bound on the volatility parameter σ such that if $\sigma \geq \bar{\sigma}$, then Condition 1 is satisfied. Let $B = 1$, $D^{\text{NP}} = 1$, and $D = 0.5$, therefore $B + D + D^{\text{NP}} = 2.5$. Suppose $\alpha_B = 0.7$, $\alpha_D = 0.5$, $\alpha_E = 0.25$, and $\Phi = 0.04$, implying that V_D as given in Eq. (2) is 2.013. Suppose, in addition, $T_L = 10$, $\pi^M(T_L) = 0.2$, $\delta = 0.1$, and $r = 0.07$. Then using the formula for $F(\cdot, \cdot)$

in Eq. (5) implies that $\sigma = 0.006$ (or 0.6%). If, for instance, $\sigma = 0.12$, then $V_R = 4.43$, and therefore at τ_P Condition 1 is satisfied. If $\delta = 0.07$, then $\sigma = 0.05$.

The intuition underlying the example is quite simple: for a given probability of default, the rating boundary will be higher with a higher volatility parameter. A lower bound on volatility therefore ensures that the rating boundary exceeds the face value of all debt claims. If the drift of the asset value is lowered, then a higher threshold volatility will be required to ensure that, at the rating boundary, asset value exceeds the face value of debt.

Define the function $v_{\tau_P}(S)$ to be the value of the claims of a coalition of players S , where $S \subset I$, I is the set of all the players of the game (firm and putable bondholders), and τ_P denotes the time at which the bonds become putable, when all players make coalitionally optimal decisions. I stretch notation slightly to let $v_{\tau_P}(D_S)$ be the market value of a set S of putable bondholders collectively owning debt with face value D_S , and $v_{\tau_P}(E \cup D_S)$ is the market value of the equity holders and a set S of putable bondholders collectively owning junior putable debt of D_S .

If putable bondholders do not put at τ_P , then because the option expires at τ_P (Assumption 4), their claims are jointly worth $v_{\tau_P}(D_P) = P_D \cdot D_P$. Now, suppose they decide to put their bonds at τ_P ; then their payoffs are given by

$$v_{\tau_P}(D_P) = D_P \quad \text{when } D_P \leq \mathcal{T} \quad (13)$$

and

$$= \frac{D_P}{D} D_F \quad \text{when } D_P > \mathcal{T}. \quad (14)$$

Clearly, when $P_D > 1$, the optimal choice for each collection of bondholders, irrespective of the decisions of the others, is not to put. When $P_D < 1$, each bondholder's payoff depends on the decisions of others, and there is no dominant strategy. When the total amount of debt that is put, D_P , is less than the trigger amount, \mathcal{T} , these debtholders recover amount D_P . When the amount put exceeds \mathcal{T} , the firm defaults and the value of the put debt is $(D_P/D)D_F$. However, if Condition 1 holds, then $D_F = D$, and each bondholder has a dominant strategy of putting. In the remainder of this section, I assume that $P_D < 1$ and Condition 1 holds at τ_P and therefore $v_{\tau_P}(D_P) = D_P$.

Define the function

$$\bar{v}_{\tau_P}(V_R, V_D; \tau_P, T) = V_{\tau_P} - \mathcal{B}(V_R, V_D; \tau_P, T) - B - D^{\text{NP}} P_D(V_R, V_D; \tau_P, T), \quad (15)$$

where \bar{v}_{τ_P} is the size of the 'pie' to be shared between the equity holders and the putable bondholders. The total value to divide between the firm and the putable bondholders is given by the value of the assets less the expected costs

of bankruptcy and the payments to senior and junior nonputable debtholders. The market value of the claims of equity holders and the set of cooperating bondholders is given by

$$v_{\tau_p}(E \cup D_C) = \bar{v}_{\tau_p} - \mathcal{L} + X_C = \bar{v}_{\tau_p} - D_P \quad \text{when } D_P \leq \mathcal{T} \quad (16)$$

and

$$= E_F + \frac{D_C}{D} D_F \quad \text{when } D_P > \mathcal{T}. \quad (17)$$

$\mathcal{L} = D_P + X_C$ is the size of the bailout loan needed by the firm immediately, and X_C is the payment for the options. When $D_P > \mathcal{T}$, a sufficient number of noncooperating bondholders put their bonds and force the firm into filing for an immediate reorganization. D_F and E_F are given in Eqs. (10) and (11).

Lemma 2. Suppose $P_D < 1$ and Condition 1 holds at τ_p , then $\bar{v}_{\tau_p} > E_F + D_F$. Therefore, the value of resources to be shared by equity holders and putable bondholders if they successfully negotiate a settlement is larger than if the firm defaults immediately.

Proof. The amount to be shared by equity holders and putable bondholders if a settlement is successfully negotiated is given by $\bar{v}_{\tau_p} = V_R - \mathcal{B} - B - D^{\text{NP}} P_D$, where \mathcal{B} is in Eq. (7). The amount to be shared by these players in an immediate reorganization is given by $E_F + D_F = (1 - \Phi)V_R - B_F - D_F^N$, which equals $(1 - \Phi)V_R - B - D^{\text{NP}}$ under Condition 1. Because $P_D < 1$, and current bankruptcy costs of ΦV_R exceed $\mathcal{B} = \Phi V_D \cdot G(\cdot, \cdot)$, the amount in continuation is strictly larger than the amount in an immediate default.

The result holds because bankruptcy costs are proportional to the value of the firm's assets at bankruptcy. If the value of assets currently is above V_D , then expected future costs of bankruptcy (hitting the threshold at some future date) are lower than current costs. Also, junior debt is priced lower in continuation because Condition 1 guarantees full recovery in immediate default.

Condition 1 is sufficient for the existence of a solution to the bargaining problem between the putable bondholders and the firm (to be discussed below). Lemma 2 implies that the putable bondholders and the firm have a larger pie to share by negotiating a settlement than they do in an immediate reorganization. It will be shown that, if the condition holds, then the value of pure bondholder coalitions is also monotonically increasing in the size of the coalition. Conversely, if $D_F < D$, then $v(\mathcal{T} + \varepsilon) = [(\mathcal{T} + \varepsilon)/D]D_F < \mathcal{T} = v(\mathcal{T})$ for a small $\varepsilon > 0$. Therefore, without the condition, the value to be shared by bondholders can decline in the number of bondholders, a situation that leads to negative marginal values of bondholders in some coalitions, and likely to disagreements of value.

4. Intrinsic and strategic values of the options at the time of the crisis

In this section, I solve the bargaining game between the firm and puttable bondholders that ensues at τ_p , the first time V hits the rating boundary V_R that permits exercise of the put options on puttable junior debt D . I shall continue to assume (unless explicitly stated) that $P_D < 1$, that is, the put options are in-the-money at τ_p . To illustrate the analysis in a simple way, the value of the pie, \bar{v}_{τ_p} , shall be assumed to be an exogenously specified constant. In the following section, I price the strategic values of the put options prior to the crisis, and then I endogenously determine \bar{v}_{τ_p} at τ_p to calibrate the model to match default probabilities of different credit rating categories. The solution concept to solve the bargaining problem between the firm and puttable bondholders uses recent results on N-player, sequential, perfect information, *non cooperative* games by Hart and Mas-Colell (1996). The game is played following a multilateral meeting with the requirement of unanimity for agreement.

Following Hart and Mas-Colell (1996), we define the function $\bar{R}_{\tau_p}(\cdot)$ that assigns a subset $\bar{R}_{\tau_p}(S)$ of \mathcal{R}^S to every coalition $S \subset I$. When all players maximize expected value, then the set $\bar{R}_{\tau_p}(S) = \{c \in \mathcal{R}^S: \sum_{i \in S} c^i \leq v_{\tau_p}(S)\}$, where $v_{\tau_p}(S)$ is defined in Eqs. (13) to (17), and it equals the expected values of the securities owned by the players in S .

The N-person noncooperative game is defined as follows. In each round there is a set $S \subset I$ of ‘active’ players and a ‘proposer’ $i \in S$. In the first round $S = I$. The proposer is chosen at random out of S , with all players in S being equally likely to be selected. The proposer makes a ‘proposal’ that is feasible, i.e., a payoff vector in $\bar{R}_{\tau_p}(S)$. If all the members of S accept it – they are asked in some pre-specified order – then the game ends with these payoffs. If the proposal is rejected by even one member of S , then I move to the next round where, with probability ρ , the set of active players is again S , and with probability $1 - \rho$, the proposer ‘drops out’ and the set of active players becomes $S - \{i\}$. In the latter case the dropped-out proposer i gets a final payoff of zero (Hart and Mas-Colell, 1996, p. 360).

The key modeling aspect is the specification of what happens if there is no agreement and the game moves to a new stage. In the framework, the breakdown of negotiations is not an ‘all or nothing’ matter. When a player leaves the game, the rest continue bargaining, albeit over a diminished pie. The cost of delay in agreement is present in the form of the breakdown probability $1 - \rho$.

Result 1. If (1) all players are expected market value maximizers, (2) Condition 1 holds, (3) the value function $v_{\tau_p}(S)$, $S \subset I$, is defined in Eqs. (13) to (17), and, (4) players follow the sequence of moves as described above, then the proposals made in the unique, stationary subgame-perfect equilibrium of the game are accepted; the equilibrium payoff vector converges to the Shapley value of the game as $\rho \rightarrow 1$. The

Shapley value of player i , if the bonds become puttable at time τ_p , is

$$\phi_{\tau_p}^i = \sum_{S \subset I} \gamma(N, S)(v_{\tau_p}[S] - v_{\tau_p}[S - \{i\}]), \tag{18}$$

where

$$\gamma(N, |S|) = \frac{(|S| - 1)!(N - |S|)!}{N!}.$$

The proof is in the appendix. The Shapley value is the average marginal value of a player, where the average results by imagining the random formation of a coalition of all the players, starting with a single member and adding one player at a time. Each player is then assigned a marginal contribution accruing to the coalition at the time of his admission. In this process of computing the expected value for an individual player, all coalition formations are considered equally likely. In Result 1, the random assignment of the first player, and the small probability of each player dropping out, leads to the possibility of each coalition forming with equal probability.

Let the bonds become exercisable at time τ_p . The *intrinsic value* of the put option is then

$$IV_{\tau_p} = \max\{1 - P_D, 0\}. \tag{19}$$

Similarly, I define the strategic value per dollar of put option owned by player i to be

$$SV_{\tau_p}^i = \left(\frac{\phi_{\tau_p}^i}{D_i} - P_D \right) \cdot \mathbf{1}_{P_D \leq 1}, \tag{20}$$

where $\mathbf{1}$ is the indicator function. As explained in the discussion following Eqs. (13) and (14), puttable bondholders only credibly threaten to put when $P_D \leq 1$.

In the following lemma, I show that if Condition 1 is also satisfied, then the strategic value of the option at the time of the put exceeds its intrinsic value.

Lemma 3. Under Condition 1, $\phi_{\tau_p}^{D_i}/D_i \geq 1$. Therefore, the strategic value of the put option of a bondholder owning puttable debt D_i exceeds its intrinsic value.

The proof is in the appendix. Intuitively, Condition 1 ensures that bondholders recover the face value of their debt by putting, even if the firm is forced into bankruptcy. Therefore, the marginal value of an additional bondholder to an existing pure bondholder coalition always equals the face value of his debt. Without the condition, the marginal value could indeed be negative, as an additional bondholder could reduce the payoff for all members of the coalition. In coalitions that include the firm, the marginal value of a pivotal bondholder, one without whom the firm is bankrupt and with whom the firm is solvent,

exceeds the face value of his bonds, and equals the face value for a nonpivotal bondholder. Therefore, under Condition 1, the marginal contribution of an additional bondholder to all possible coalitions is always larger than the face value of his debt.

The result in Hart and Mas-Colell is a natural extension of the classic two-player alternating-offers bargaining game in Rubinstein (1982). For more than two players, the Rubinstein model yields a folk-like theorem (a continuum of equilibria) if the solution concept is merely subgame perfection and, thus the additional refinement of stationary equilibrium must be imposed (Sutton, 1986, p. 722) Convergence of the bargaining solution to the Shapley value has also been obtained under similar sets of rules and alternative meeting arrangements of players in Harsanyi (1981), Gul (1989), and Winter (1994). Due to its robustness to different modelling nuances, the Shapley value is increasingly being used in applications by several authors as a solution to symmetric information multilateral bargaining situations (see, e.g. Aivazian and Callen, 1983; Bergman and Callen, 1991; Hart and Moore, 1990; Rajan and Zingales, 1998).

The alternating-offers framework implies that the strategic value is more evenly distributed among the players than in strategic models in which assigned player(s) can make take-it-or-leave-it offers. As an illustration, consider the sinking fund game between bondholders and the firm in Dunn and Spatt (1984), in which the bondholders move first and simultaneously. When the sinking fund requirement exceeds the active supply of bonds, each bondholder sells the bonds to the firm at its face (par) value even though the traded price per dollar of face value of the bond is $P < 1$. Because the pie to be shared between the firm and any bondholder is $(1 - P)$ per dollar of face value, in the alternating-offers framework it can be shown that irrespective of the distribution of debt, the firm buys back the bonds at a price of $(1 + P)/2$, exactly half the pie. The choice of the rules of the game depends on institutional arrangements but is also determined by the costs imposed on the firm by bondholders. In the sinking fund market, the lower cost setting in Dunn and Spatt (1984), in which bondholders need simply submit their bids by mail to the firm, is more appropriate given that the payout to the firm is bounded by $(1 - P)$ times the face value of debt owned by active bondholders, likely a small fraction of the value of the firm's assets. In the puttable securities crises in this paper, the costs of setting up an alternating-offers bargaining game are likely larger, but incurring them is also appropriate because of the potential bankruptcy costs confronted by the firm.

4.1. Equally sized bondholders

When there are N bondholders each holding D/N of the face value of puttable debt, the notation is further simplified by defining $v_{\tau_p}(E \cup D_n)$ as the value of securities that can be distributed between the equity holders and n cooperating

bondholders. Using Eqs. (16) and (17)

$$v_{\tau_p}(E \cup D_n) = \bar{v}_{\tau_p} - \frac{N-n}{N}D \quad \text{when } \frac{N-n}{N}D \leq \mathcal{T} \tag{21}$$

$$= E_F + \frac{n}{N}D_F \quad \text{when } \frac{N-n}{N}D > \mathcal{T}. \tag{22}$$

Using Eqs. (13) and (14) for pure bondholder coalitions,

$$v_{\tau_p}(D_n) = \frac{n}{N}D \quad \text{when } \frac{n}{N}D \leq \mathcal{T} \tag{23}$$

$$= \frac{n}{N}D_F \quad \text{when } \frac{n}{N}D > \mathcal{T}. \tag{24}$$

There are $N + 1$ players in the game, with N bondholders. From Eq. (18), the Shapley value of the firm for this special case is

$$\begin{aligned} \phi_{\tau_p}^{E,N} &= \sum_{n=0}^N \sum_{C:|C|=n} \gamma(N+1, n+1)(v_{\tau_p}(E \cup D_C) - v_{\tau_p}(D_C)) \\ &= \frac{1}{N+1} \sum_{n=0}^N (v_{\tau_p}(E \cup D_n) - v_{\tau_p}(D_n)) \end{aligned} \tag{25}$$

since there are $N!/(N-n)!n!$ coalitions with n bondholders.

Result 2. The Shapley value of the equity holders with N equally sized bondholders at τ_p when Condition 1 holds and $P_D < 1$ is given by

$$\phi_{\tau_p}^{E,N} = E_F \frac{N - M^c + 1}{N + 1} + \frac{M^c}{N + 1} [\bar{v}_{\tau_p} - D]; \tag{26}$$

$M^c(\mathcal{T}, D, N) = \lfloor N\mathcal{T}/D \rfloor + 1$ is the smallest number of bondholders that can force the company into a reorganization. The Shapley value of each bondholder is given by $\phi_{\tau_p}^{D/N,N} = \frac{1}{N}(\bar{v}_{\tau_p} - \phi_{\tau_p}^{E,N})$, and the strategic value of the put option per dollar equals

$$SV_{\tau_p}^{D/N,N} = \frac{N - M^c + 1}{N + 1} \frac{\bar{v}_{\tau_p} - E_F}{D} + \frac{M^c}{N + 1} - P_D. \tag{27}$$

$M^c/(N + 1)$ represents the proportion of all coalitions (including the firm) in which the firm reaches a settlement and avoids bankruptcy.⁴ When there are

⁴ $\lfloor N\mathcal{T}/D \rfloor$ is the largest integer less than or equal to $N\mathcal{T}/D$. Note that $\lfloor N\mathcal{T}/D \rfloor + 1$ is not always equal to $\lceil N\mathcal{T}/D \rceil$. For example when $\mathcal{T}/D = \frac{2}{3}$ and N equals 3, the former number equals 3 and the latter 2. However, $D\frac{2}{3} = \mathcal{T}$, therefore the smallest threatening coalition must have three bondholders.

N equally sized bondholders, it is a sufficient statistic for the bargaining strength of the firm relative to the puttable bondholders. Since the fraction is nonmonotonic in the number of puttable bondholders, it leads to a nonmonotonicity in the bargaining strength of the firm. I illustrate the phenomenon with an example depicted in Fig. 1.

Example 2 (Nonmonotonicity of strategic value in N). The following parameters describe the problem at the time the bonds first become puttable: $\bar{v} = 3$, $D = 0.5$, $D_F = 0.5$, $\mathcal{F} = 0.1$, $E_F = 2$, and $P_D = 0.9$. Using Lemma 2, I calculate the strategic value of the firm, the strategic value of each bondholder,

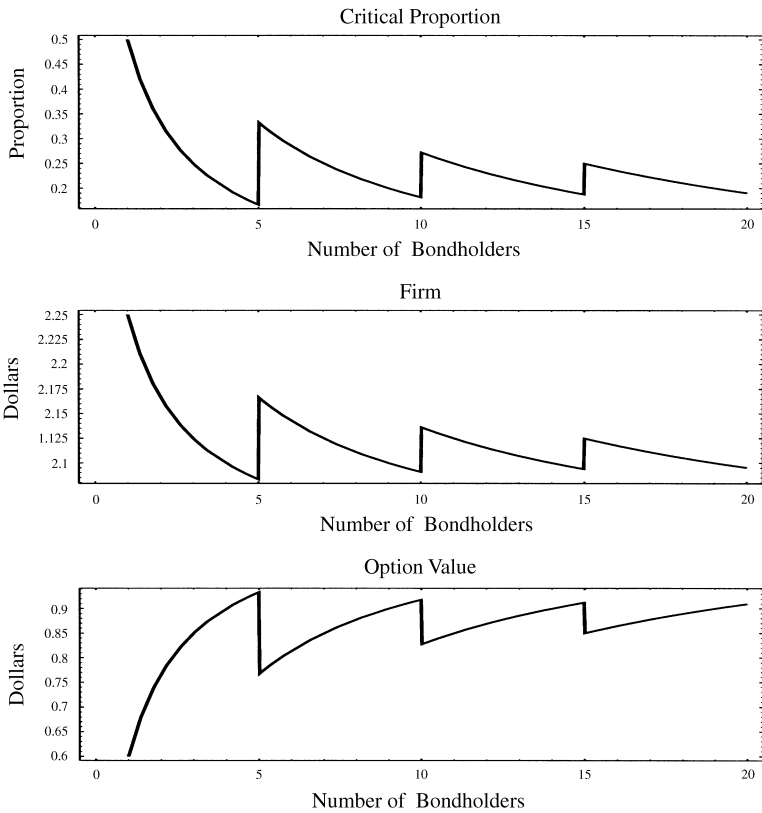


Fig. 1. The nonmonotonicity of strategic value in the number of equally sized bondholders. The proportion in the top panel equals $M^c/(N + 1)$; it represents the proportion of all coalitions (including the firm) in which the firm reaches a settlement and avoids bankruptcy. The strategic value of the players is called as shown in Lemma 2. The strategic value of the option is defined in Eq. (20). The following parameters describe the problem: $\bar{v}_{cr} = 3$, $D = 0.5$, $D_F = 0.5$, $T = 0.1$, $E_F = 2$, and $P_D = 0.9$.

and the payment for the option as defined in Eq. (20) for $N = 1, \dots, 20$. The three values, along with the ratio $M^c/(N + 1)$, are displayed in the three panels of Fig. 1. As seen in the figure, the strategic values of the firm and the bondholders vary positively and negatively, respectively, with the ratio. The option value can range between 0.6 and 0.9 as N varies between one and 20. Therefore, the *discreteness* of bondholders plays an important role in the determination of strategic values. The ratio $M^c/(N + 1)$ tends to $\mathcal{F}/D = 0.2$ as $N \rightarrow \infty$, thus providing a limiting value for the value of the firm and the strategic value of the options.

Corollary 1. The limit of the equity holders' value and the strategic value of the put option per dollar debt are

$$\phi_{\tau_p}^{E, \infty} \equiv \lim_{N \rightarrow \infty} \phi_N^E = E_F \left(1 - \frac{\mathcal{F}}{D} \right) + \frac{\mathcal{F}}{D} [\bar{v}_{\tau_p} - D], \tag{28}$$

$$SV_{\tau_p}^{\infty} = \lim_{N \rightarrow \infty} \frac{1}{D} \sum_{i=1}^N \phi_{\tau_p}^{D/N, N} - P_D = \left(1 - \frac{\mathcal{F}}{D} \right) \frac{(\bar{v}_{\tau_p} - E_F)}{D} + \frac{\mathcal{F}}{D} - P_D. \tag{29}$$

The strategic value of the put options exceeds the intrinsic value, even when there are an infinite number of bondholders, as long as $\mathcal{F} < D$.

The proof follows by taking the limits of the values on Result 2. Because $\bar{v}_{\tau_p} - E_F > D$ under Condition 1, the strategic value of each bondholder exceeds his intrinsic value.

The form of the limiting bargaining value of the equity holders suggests that \mathcal{F}/D can be interpreted as a probability, although, it is really only a pseudo-probability because, in equilibrium, the first proposer makes an offer that all players accept. It corresponds to the probability of a coalition forming in which the firm need not file for a reorganization when coalition formation is viewed as being random under the rules of the bargaining game defined in Section 4. With infinitesimal bondholders there is a probability $1 - \mathcal{F}/D$ of a coalition forming that will force the company into a reorganization. With the complementary probability \mathcal{F}/D , the company will purchase the options back from enough bondholders to avert a reorganization of the firm. Put slightly differently, the Shapley value of a player is the average marginal contribution of the player across all possible coalitions. When each bondholder contributes an infinitesimal amount, the average and marginal contributions coincide, and the Shapley value of each bondholder is simply his average contribution. Ostroy (1984) provides an elucidation of this point.

It is well known that as the number of players increases to infinity, the Shapley value converges to the Walrasian equilibrium of the game (see, e.g., Debreu and Scarf, 1963; Wooders and Zame, 1984). The limiting value obtained above is useful because it enables the pricing of covenants of debt in a perfectly

competitive setting. It is seen that even with infinitesimal putable bondholders in a *decentralized* setting, such bondholders have joint threatening power. The equity holders must be concerned about putable bonds outstanding, even if they are held by infinitesimal bondholders, because if each bondholder unilaterally puts his bond, the firm will be forced into bankruptcy. The limiting value is also useful as a convenient ‘back-of-the-envelope’ calculation of value for put options held publicly by a large number of small and equally sized bondholders.

4.2. The bargaining strength of a large bondholder

Large bondholders are commonly observed in lending arrangements, often performing various tasks in servicing the loan for other lenders. While it seems that a larger player should have a greater threatening potential on the firm’s limited liquid assets, it is not clear if the value per dollar of debt owned is indeed larger. Indeed, in the simple case with only one large and one small bondholder, it can be shown that the large bondholder has a higher strategic value per dollar only if the amount of the large bondholder’s putable debt exceeds the trigger amount *and* the amount held by the small bondholder is smaller than the trigger amount. When there are several smaller bondholders, I show in this section that the roles played by smaller bondholders in the presence of a large bondholder can make their threatening powers *complementary* or *substitutable* depending on the trigger amount of debt relative to the distribution of debt, thus affecting their strategic valuation relative to the larger player. If they act as strategic complements, then their collective value should increase as their number increases, and the reverse for substitutes. I show by an example that either sort of behavior can emerge.

Consider the potential coalitions that can be formed with N bondholders of equal size collectively holding debt D , a large bondholder with debt D^1 , and the equity holders. Throughout, $M^0(\mathcal{T}, D, N) = \min\{\lfloor (\mathcal{T}/D)N \rfloor + 1, N\}$ and $M^1(\mathcal{T}, D, D^1, N) = \max\{\lfloor [(\mathcal{T} - D^1)/D]N \rfloor + 1, 0\}$. M^0 is the smallest number of equally sized bondholders that can collectively threaten the firm. M^1 is the smallest number of equally sized bondholders that together with the large bondholder can put their bonds and force the firm into a reorganization. Obviously, $M^1 < M^0$. Lemma 4 in the appendix provides expressions for each player’s value, similar to those in Result 2, with M^0 and M^1 playing the role of M^c .

Example 3 (Smaller bondholders can be strategic substitutes or strategic complements). Suppose there are one large and several small bondholders collectively holding D^1 and D of putable debt, respectively. The following parameters describe the problem: $\bar{v}_{\tau_p} = 5$, $D = 0.6$, $D^1 = 0.3$, $\mathcal{T} = 0.1$, and $E_F = 3$. Using Lemma 4, Fig. 2 shows the strategic values for all players for three different cases. In Case (1), $\mathcal{T} < D^1 < D$; therefore, a collection of at least

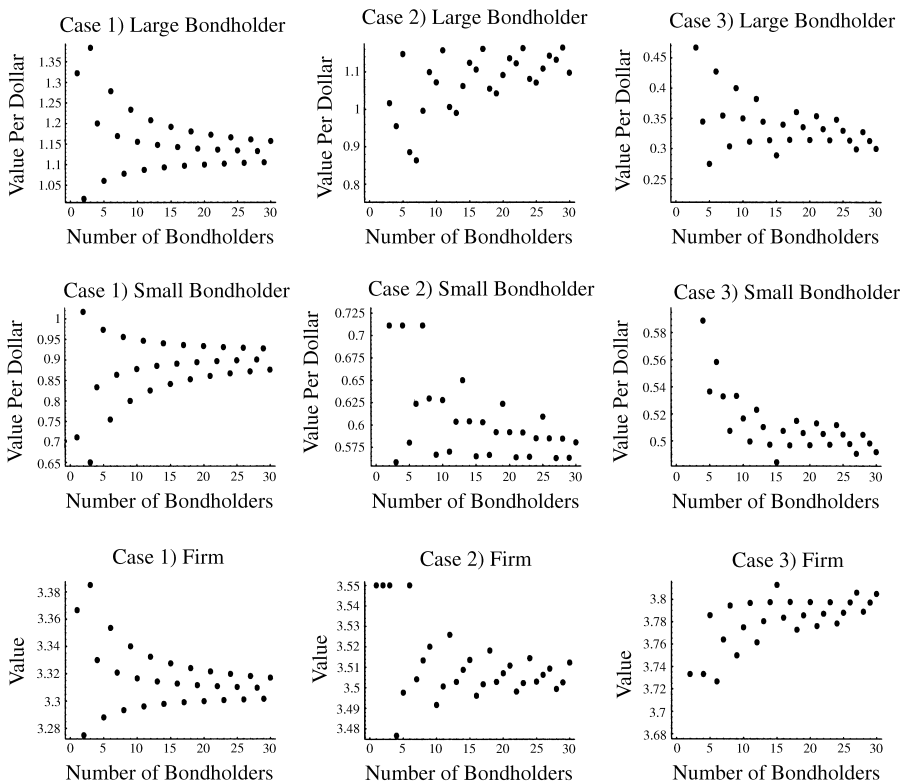


Fig. 2. One large and several small bondholders: Are small bondholders strategic complements or substitutes? The strategic value of the players is calculated as shown in Lemma 4. The strategic value of the option is defined in Eq. (20). The following parameters describe the problem: $\bar{v}_{\tau_p} = 5$, $D = 0.6$, $D^1 = 0.3$, $E_F = 3$, and $P_D = 0.9$. The figures show the values of the players as the trigger level is increased from 0 to $D + D^1$. The number of small bondholders is allowed to vary between one and 30. Three separate cases are considered: in Case (1) $\mathcal{T} = 0.2$, in Case (2) $\mathcal{T} = 0.4$; and in Case (3) $\mathcal{T} = 0.7$.

M^0 bondholders (defined above), the large bondholder alone, or the large bondholder and M^1 bondholders can credibly threaten the firm. As the debt D is divided among a large number of bondholders, a larger number of threatening coalitions can be formed. Therefore, as N increases, the value of the large bondholder relative to small bondholders tends (the nonmonotonicity as discussed previously applies) to decrease. Accordingly, the value of the firm tends to decline as the proportion of threatening coalitions grows. In Case (3) with $D^1 < D < \mathcal{T}$, the large bondholder cannot trigger a bankruptcy on his own. Cooperation with some smaller bondholders is needed. As N increases, the potential threatening coalitions he is a part of increases, therefore increasing his

value with the number of small bondholders. However, as the debt of smaller bondholders is divided up among a larger number, some coalitions of smaller bondholders cease to threaten, lowering the proportion of threatening small bondholders, and therefore increasing the value of the firm. Case (2) with $D^1 < \mathcal{F} < D$ is the intermediate situation in which there is no clear trend but the discreteness of bondholders (discussed following Example 2) can cause large fluctuations in value as the number of small bondholders increases.

Overall, in Case (1) smaller bondholders complement each others' threats while in Case (3) they tend to be substitutes. With an increase in the number of small bondholders, therefore, the value of the firm decreases in Case (1) and increases in Case (3). Also the ratio of the large to small bondholders' strategic option value declines in Case (1) and increases in Case (3).

4.3. Strategic values of reset bondholders

So far I have assumed that if more debt than permitted by the trigger is put, the company is forced into bankruptcy. Some other scenarios are also relevant: many debt contracts contain restrictive covenants that are triggered if financial ratios such as the cash-to-interest payments ratio move out of a specified range, or after a major recapitalization, or at a pre-committed date, at which point the coupon is reset so that the debt is priced at par. Let δ be the payout ratio on all liabilities (Assumption 1). Further, assume that a reset on more than the face value of \mathcal{F} bonds will, at the firm's current payout rate, require new injections (negative dividends) of funds from equity holders. Absent their willingness to commit new funds, the total payout ratio will have to be raised to δ' . As seen in Eq. (7), a higher payout ratio lowers the drift of the asset process, and increases the likelihood of the firm defaulting and the expected discounted costs of future bankruptcy.

Suppose the firm has committed to reset the coupon on the junior debt, D , at a pre-committed date, τ_R . I allow for the possibility that the bond trades at a price $P_D \leq 1$ despite the reset coupon (usually calculated with the assistance of investment bankers). As in Section 3 define the functions

$$\bar{v}_{\tau_R}^1(V, V_D; \tau_R, T, \delta) = V - \mathcal{B}(V, V_D; \tau_R, T, \delta) - B, \quad (30)$$

$$\bar{v}_{\tau_R}^2(V, V_D; \tau_R, T, \delta') = V_{\tau_R} - \mathcal{B}(V, V_D; \tau_R, T, \delta') - B, \quad (31)$$

where $\bar{v}_{\tau_R}^i$, $i = 1, 2$, is the size of the pie to be shared between the equity holders and reset bondholders when the payout ratio is δ and δ' , respectively. Reset bondholders have two choices at the date of the reset; they can either hold on to their bonds, which will be valued at P_D after the reset, or sell them back to the firm at a negotiated price. Using similar notation to that in Section 4, let D_P be the face value of debt held by bondholders who choose the first alternative, and

let D_C be the face value of those choosing the latter alternative. The market value of the assets of a collection of pure bondholders owning face value of resettable debt D_P equals $v_{\tau_r}(D_P) = D_P P_D$. Similarly,

$$v_{\tau_r}(E \cup D_C) = \bar{v}_{\tau_r}^1 - D_P P_D \quad \text{when } D_P \leq \mathcal{T} \quad (32)$$

$$= \bar{v}_{\tau_r}^2 - D_P P_D \quad \text{when } D_P > \mathcal{T}. \quad (33)$$

Once again, let $M^c(\mathcal{T}, D, N) = \lfloor N\mathcal{T}/D \rfloor + 1$ be the smallest number of reset bondholders that can force a larger payout ratio. When there are N equally sized bondholders, similar to Result 2, we obtain the strategic values of the equity holders and the value per dollar of resettable debt as

$$\phi_N^E = \frac{N - M^c + 1}{N + 1} \bar{v}_{\tau_r}^2 + \frac{M^c + 1}{N + 1} \bar{v}_{\tau_r}^1 - D P_D, \quad (34)$$

$$\frac{\phi^D}{D} = P_D + \frac{M^c + 1}{N + 1} \frac{\bar{v}_{\tau_r}^1 - \bar{v}_{\tau_r}^2}{D}, \quad (35)$$

and whenever $\mathcal{T} < D$, the firm will repurchase at least $D - \mathcal{T}$ bonds at their strategic value, which exceeds P_D . In addition to the traded price of the bond, the strategic value includes a fraction of the increase in the discounted expected future costs of financial distress, which result if the payout rate of the firm is increased. As before, we find that the strategic value depends on the distribution of resettable debt among bondholders, and even as $N \rightarrow \infty$, the incremental strategic value is nonvanishing. Special cases with different-sized bondholders also lead to similar results.

One such situation occurred in the spring of 1990, when RJR Nabisco was forced to repurchase a substantial amount of its outstanding \$7 billion of reset bonds.⁵ Market participants at the time worried that the reset would force RJR into choosing an extraordinarily large coupon that would increase the likelihood of driving the firm into bankruptcy (see, e.g., *The Wall Street Journal*, June 28, 1990, p. A1; Perold, 1996). Consistent with the analysis in this subsection, RJR bought back a sufficient amount of such debt at a price substantially above its traded price, just prior to negotiations.

5. Pricing the strategic and intrinsic value prior to exercise

In this section, I price the intrinsic and strategic value of the put options prior to the onset of the crisis described in Section 4, using the payoffs of the put

⁵ I thank the JFE referee for making the analogy between puttable securities and the RJR situation.

options Eqs. (19) and (20), respectively, as boundary conditions. As shown in the previous sections, the strategic value of bondholders depends on the asset value of the firm at the time of exercise; embedding the strategic valuation in a structural framework of bond pricing provides an endogenous calculation of the asset value at the rating (put-triggering) boundary, and therefore provides calibrated values for poison puts of firms with different ratings. At the end of this section, I compare the yield reductions from poison put bonds relative to nonputable bonds under intrinsic and strategic valuation, and compare them with yield reduction in secondary markets for investment-grade bonds in 1989, as documented in Crabbe (1991).

The intrinsic value of the option prior to the crisis at time t is

$$P_t^{IV} = E[e^{-r(\tau_p - t)} \max\{1 - P_D(V_R, V_D; \tau_p, T), 0\}], \quad (36)$$

in which τ_p is the hitting time of V to V_R as described in Assumption 4 and expectations are taken with respect to the risk-neutral measure. To further simplify the analysis, I impose another condition on permissible parameter values for the problem.

Condition 2. $P_D(V_R, V_D; t, T)$ is increasing in t . The condition ensures that there exists a T^* satisfying $0 \leq T^* \leq T$ such that $P_D(V_R, V_D; \tau_p, T) < 1$ whenever $\tau_p < T^*$ and $P_D(V_R, V_D; \tau_p, T) \geq 1$ for $\tau_p \geq T^*$. Leland and Toft (1996, Fig. 3) show that the condition will hold for a ‘wide’ range of parameters and is violated when the riskless rate is very high relative to the volatility of the asset process. When Condition 2 holds, bondholders credibly threaten to put if $\tau_p \leq T^*$, and the option expires unexercised for $\tau_p > T^*$ [see the discussion following Eq. (14)]. Without the additional condition, the valuation of the put option is still fairly straightforward, the only qualification being that the put option will be exercised in a disjoint subset of $[0, T]$.

Specializing my analysis to cases in which Condition 1 and Condition 2 hold, the intrinsic value of the option at time t is

$$\begin{aligned} P_t^{IV}(V_t, V_R; t, T^*) &= P^I(V_t, V_R; t, T^*) - P^{P^b}(V_t, V_R; t, T^*) \\ &\equiv G(V_t, V_R; t, T^*) \\ &\quad - \int_0^{T^*} f(V_t, V_R; t, \tau_p) e^{r(\tau_p - t)} P_D(V_R, V_D; \tau_p, T) d\tau_p, \end{aligned} \quad (37)$$

in which $P^I = G(V_t, V_R; t, T^*)$ is the time- t value of a dollar at τ_p for all $0 \leq \tau_p \leq T^*$, $P_t^{P^b}$ is the time- t value of a dollar of nonputable debt at such τ_p , and $f(\cdot, \cdot)$ and $G(\cdot, \cdot)$ are the functions defined in Eqs. (4) and (6), respectively. The formulas simply follow by taking expected values using the density of the hitting time and discounting at the riskless rate. To price the strategic value of

the put option, I need the prices of two other securities: using Eq. (15),

$$\begin{aligned}
 P^{\bar{v}}(V_t, V_R; t, T^*) &= P^1(V_t, V_R; t, T^*)(V_R - B) \\
 &\quad - \int_0^{T^*} f(V_t, V_R; t, \tau_P) e^{-r(\tau_P - t)} \cdot (\Phi V_D \cdot G(V_R, V_D; \tau_P, T) \\
 &\quad + D^{\text{NP}} P_D(V_R, V_D; \tau_P, T)) d\tau_P,
 \end{aligned}
 \tag{38}$$

which is the value of the discounted expected value of the pie in successful negotiations at τ_P . Similarly, using Eq. (11),

$$P_t^{E_f} = P_t^1 \cdot ((1 - \phi)V_R - D^{\text{NP}} - D),
 \tag{39}$$

the value of the firm if negotiations break down at τ_P . The strategic value of the put options for any of the cases in Sections 4.1 and 4.2 can be calculated using the four prices $P_t^{\bar{v}}$, $P_t^{E_f}$, P_t^1 , and $P_t^{P_D}$. For example, when there is a continuum of equally sized bondholders, then using Eq. (29),

$$P_t^{SV} = \left(1 - \frac{\mathcal{F}}{D}\right) \frac{P_t^{\bar{v}} - P_t^{E_f}}{D} + \frac{\mathcal{F}}{D} P_t^1 - P_t^{P_D}.
 \tag{40}$$

I provide a simple example to illustrate the pricing.

Example 4 (Investment-grade credit rating, the value of options, and yield reduction from poison puts). I continue to use the parameters in Example 1 with $r = 0.07$ and $c_D = 0.072$ and, in addition, I assume that there is a continuum of equally sized bondholders. The parameters imply that the firm’s default boundary $V_D = 2.013$, and the rating boundary that satisfies $T_L = 10$ and $\pi^M(T_L) = 0.21$ (matches the average probability of a downgrade to a Ba rating in *Moody’s Investor Service, Global Credit Research, 1997* report on historical default rates) triggers the puts at $V_R = 4.44$. The maturity of the bonds is 20 years and the puts are exercised only if the rating boundary is hit prior to $T^* = 13.41$ years. Condition 1 is clearly satisfied since $V_R > D + D^{\text{NP}} + B$, and Condition 2 can be verified numerically using the closed-form expression for the bond price in Eq. (3). I further assume that $\mathcal{F} = 0.3$, which implies a liquidity constraint of 0.17. The results are shown in Table 1.

As seen in the table, both the intrinsic and strategic values decrease in value as the credit rating of the firm improves. For a Baa-rated bond, for example, the strategic value per \$100 of outstanding putable debt is about \$5.87, while its intrinsic value is only about \$1.41. For higher rated bonds, both values decrease as the asset value is raised to match the lower default probabilities, correspondingly leading to lower transition probabilities to the speculative grade boundary. The significantly higher strategic value of the options helps justify the finding by Crabbe (1991) that put option protection in the late 1980s lowered

Table 1

Credit rating, the value of options, and yield reductions from poison puts

The capital structure of the firm, $B = 1$, $D^{\text{NP}} = 1$, and $D = 0.5$, and the parameters for recoveries of different securities, $\alpha_B = 0.7$, $\alpha_D = 0.5$, $\alpha_E = 0.24$, and $\Phi = 0.04$, imply that V_D as given in Eq. (2) equals 2.013. In addition, I assume parameters $r = 0.07$, $\delta = 0.1$, $c_D = 0.072$, and $\sigma = 0.12$. The rating boundary $V_R = 4.44$ matches the ten-year probability of default for Ba-rated bonds. Bonds mature in 20 years and the parameters imply that the poison puts will be exercised if the asset value hits V_R in the time interval $[0, T^*] = 13.45$. For the calculation of strategic values, I assume $\mathcal{F} = 0.3$, and there is a continuum of bondholders. The market value of the firm's assets are in the first column. π_{20} is the 20-year probability of default starting with the asset value as shown, calculated using $F(V, V_D; 0, 20)$ in Eq. (5). This default rate is usually associated with a Moody's rating shown in the following column. w_R in the fourth column represents the proportion of investment-grade bonds outstanding (in percent) at the end of 1989 with the given credit rating; these proportions will be used to create averages for all investment-grade bonds. P^{IV} and P^{SV} are the intrinsic and strategic value of poison puts per \$100 of face value. The values of these securities are calculated using Eqs. (37) and (40). Finally, yr^{IV} and yr^{SV} are the secondary-market reductions in yields (in basis points) from comparable nonputtable bonds that result when the intrinsic and strategic value of the puts are taken into consideration.

Assets	π_{20}	Rating	w_R	P^{IV}	P^{SV}	yr^{IV}	yr^{SV}
8.9	0.118	Baa	23.3	1.41	5.87	13.3	54.8
11.2	0.051	A	40.2	0.54	2.59	4.8	23.7
12.8	0.028	Aa	26.0	0.27	1.44	2.4	12.5
13.9	0.020	Aaa	10.5	0.12	1.06	1.6	9.2

investment-grade yields by about 30 basis points in the secondary market compared with nonputtable bonds with similar ratings. Indeed, averaging (weighting each category by the proportion outstanding) the yield reductions over the four rating categories using intrinsic value results in a lowering of about 5.7 basis points, but result in a substantially larger lowering of 25 basis points using strategic value.

There are several economic assumptions that could be changed to give higher intrinsic values, but I argue that none are completely plausible. Firstly, increasing the volatility parameter will increase the values of options to match the observed yield reduction through intrinsic valuation. However, the asset values in the first column are chosen to maintain Moody's 20-year probabilities of default. If σ is increased, then the initial asset value would have to be raised and the resulting yield reduction is not affected significantly. For example, if $\sigma = 0.18$, then the asset value to maintain the π_{20} of a Baa rating, would have to be raised to 15.8. The rating boundary V_R would also be increased to 6.06, but this would increase both \bar{v}_{t_p} , and E_F , which would have offsetting effects on strategic values of bondholders. I calculate that the change in yield reduction from intrinsic valuation in this case would be less than one basis point for a 50% change in the volatility parameter. Similar logic applies for other parameters of the asset-volatility process and for the capital structure of the firm.

Secondly, it can be assumed that investors' anticipated transition probabilities from investment grade to speculative grade were higher in the late 1980s than observed over the full sample. To justify the large yield reductions, a five-fold increase in the transition probabilities would be needed. However, actual downgrades for most rating categories in the period were less than twice their normal rates (see Exhibit 9 in *Moody's Investor Service, Global Credit Research, 1997* report on rating migration), and it would be reasonable to assert that only part of entire increase was anticipated. Therefore, this explanation is also unlikely to account for a sufficient increase in the intrinsic value of the put options in the sample.

Thirdly, liquidity risk, or the risk of being unable to transact in the bond market in turbulent conditions, might explain a substantial part of the yield reduction. However, the yield reductions on puttable bonds noted by Crabbe (1991) are computed relative to nonputtable bonds with similar ratings and maturity. Because these two factors are known to be the main determinants of liquidity, it would be unrealistic to expect liquidity to significantly account for the difference in yields.

Finally, it can be argued that institutional investors preferred the protected poison puts, not for their strategic value but for the embedded safety clause that these bonds provide, and indeed, the most conservative investors would not be able to take advantage of the strategic value because of SEC regulations. Dammon et al. (1993) suggest a similar behavioral explanation as a possible explanation for the pricing differences between RJR's cash and pay-in-kind bond prices in 1990. However, for such a pricing difference to be sustained, it must be case that transactions costs prohibit the implementation of arbitrage strategies that would replicate the puttable bond at about its theoretical intrinsic value. For example, if the bond price follows a continuous path, a stop-loss order strategy holding the nonputtable bond could, within the cost of one transaction, replicate the payoff of a puttable bond. But the pricing differences (Column 6 minus Column 5 in Table 1) necessary to support the yield reduction are too large to be sustained by transactions costs of the marginal investor in the investment-grade market: for example, in the dealer market, Warga (1991) reports bid/ask spreads in 1990, a period of unusual turmoil, of 50 cents per \$100 in par value (spreads in more normal times are about one-fourth of this), substantially lower than the pricing differences. Again, it would be reasonable to assume that market participants at that time expected bid-ask spreads to decline in several months back to normal levels and to factor in lower average transactions costs for the maturity of the puttable bonds.

6. Bargaining problem at the time of Kmart's poison put crisis

In this section, I shall calibrate the model in Section 4 to Kmart's 1995 poison put crisis. The goal of the calibration exercise is to understand the implications

of the bargaining model for (1) the total amount of the settlement paid to putable bondholders, (2) the relative payments to large and small bondholders, and (3) small changes that the firm could have made in its put issuance policy that would have reduced its payout under distress.

The crucial determinant of the strategic value of the putable debtholders is their ability to induce premature financial distress in the firm, quantified in the parameter Φ in the model. Financial economists have found it difficult to measure the costs of financial distress. Estimates of the direct costs of bankruptcy have generally been 2–4% of the value of assets see, e.g., Warner, 1977; Weiss, 1990), and if the effects of ongoing economic problems are not filtered out, the costs can range as high as 20% (Andrade and Kaplan, 1998). If one assumes that an early put will induce purely financial distress, then its damage would be on the lower end, supporting the view in Haugen and Senbet (1978) that claimants in pure financial distress should be able to renegotiate without affecting the value of the underlying assets of the firm. Nonetheless, given the measurement problems, I provide strategic values for a range of bankruptcy costs, and also reverse the question to ask what level of the assumed bankruptcy cost parameter is consistent with the aggregate observed settlement.

The value of the pie to be shared between putable bondholders and the values of different claim holders in bankruptcy are based on publicly released data. From Kmart's 1995 annual report, total book assets, A_0 , are \$15.3 billion; senior debt, B , equals \$5.1 billion; junior nonputable debt, D^{NP} , equals \$4.3 billion; the putable debt held by one large bondholder, D^1 , equals, \$0.1 billion; and the putable debt held by 20 bondholders, D , equals \$0.45 billion. There were 486,511 shares outstanding at \$14.50 per share (historical stock and bond prices from *Bloomberg Financial Markets*), making the market value of equity, V_E , equal to \$7.05 billion. Based on 24 bond prices available for September 1995, the average price of the junior debt is \$0.98 per dollar of face value. The market value of all the junior debt, therefore, is \$4.75 billion. It has been assumed that senior debt is valued at par and therefore, the total market value of the firm's assets is the sum of the market value of its liabilities, or \$16.9 billion. The firm can accelerate payment on no more than $\mathcal{T} = \$110$ million of putable debt.

The parameters of the model are obtained from previous empirical studies. Following the literature on structural form pricing models, I use the point estimates of average recovery rates (see, e.g., Leland, 1994; Longstaff and Schwartz, 1995) for the constant recovery rates assumed in the model. In fact, recovery rates exhibit substantial variation across firms and time. However, because I assume that agents are risk-neutral and that these parameters are used to price claims with fairly long maturities, their unconditional expectations are a reasonable approximation for investors' expectations for recoveries in future distress situations. These parameters are not used for the recoveries in the case of an immediate bankruptcy, which are determined instead by absolute priority (Assumption 5) and are of first-order importance for the settlement of claims. In

sum, the recovery per dollar on bank loans, α_B , is \$0.70 and the recovery per dollar on junior debt, α_D , is \$0.50 (*Moody's Investor Service, Global Credit Research*, 1997 study on historical default rates). The fraction of equity value of the reorganized company received by shareholders, α_E , is 0.25 (Weiss, 1990).

Bankruptcy costs in an immediate bankruptcy are simply ΦV , and I shall examine various levels of Φ . Discounted expected future bankruptcy costs, if a current settlement is negotiated, are calculated as follows. The recovery parameters and balance sheet items jointly imply the default boundary defined in Assumption 3. Discounted expected future bankruptcy costs are then calculated using Eq. (7) by inferring the function $G(\cdot, \cdot)$ from the pricing formula in Eq. (3) for nonputtable bonds and available bond prices and using the assumed level of Φ .⁶ In contrast, current bankruptcy costs would be $\Phi \cdot \$16.9$ billion or \$338 million. Therefore, the puttable bondholders can cause a deadweight loss of about \$298 million. These calculations are repeated for other values of Φ .

Using Assumption 5, Eqs. (8) to (11) provide the values of different claimholders in an immediate bankruptcy using absolute priority. As an illustration, I show these values when $\Phi = 0.02$:

$$\begin{aligned} B_F &= \min\{(1 - 0.02) \cdot 16.9, 5.1\} \\ &= \$5.1 \text{ billion (Recovery on senior debt),} \\ (D_F^{\text{NP}} + D_F) &= \min\{(1 - 0.02) \cdot 16.9 - 5.1, 4.85\} \\ &= \$4.85 \text{ billion (Recovery on junior debt),} \\ E_F &= (1 - 0.02)16.9 - 5.1 - 4.85 \\ &= \$6.61 \text{ billion (Shareholders' recovery).} \end{aligned}$$

The size of the pie after a successful options transaction for $\Phi = 0.02$ is obtained by using (15):

$$\bar{v}_{t_p} = 16.9 - 0.04 - 5.1 - 4.3 \cdot 0.98 = \$7.54 \text{ billion.}$$

⁶ For example, when $\Phi = 0.02$, then $V_D = 7.033$, and $G(V, V_D; 0, 28)$ for 28-year bonds equals 0.293; therefore, these costs are about \$40 million. Using bond pricing data from *Bloomberg Financial Markets*, I find that Kmart's bonds with remaining maturity of 28 years (the longest maturity bonds outstanding in September 1995) with a coupon of 7.95% and a face value of \$100 traded at \$94.21, and those with a coupon of 8.375% traded at 98.25 just prior to the start of negotiations. The two prices and (3) imply two equations in the two unknowns $F(V, V_D; 0, 28)$, and $G(V, V_D; 0, 28)$. I note that bankruptcy costs are proportional to $G(V, V_D; 0, \infty)$. However, because $G(\cdot, \cdot)$ is essentially an integral of densities discounted at a riskless rate of 0.07, the part of the integral in excess of 28 years has a negligible effect. In addition, because $G(\cdot, \cdot)$, which depends on σ_V , δ , and r , is multiplied by Φ , errors due to specification of the asset-value process, and other model and estimation errors, will have a second-order effect on the estimation of discounted expected future bankruptcy costs.

Table 2

Bankruptcy costs and strategic value of poison put bondholders in the Kmart case

All values are in billions of dollars. The parameters of the problem are as calibrated to Kmart's financial crisis in September 1995; the details are in Section 6. The first column shows an assumed level of the proportional bankruptcy cost parameter and is used for calculating the value in all the following columns. The second column shows the pie to be shared between equity holders and the puttable bondholders if negotiations are successful, calculated using Eq. (15). The third column shows the value of the equity holders in an immediate bankruptcy, calculated using Eq. (11), the fourth and fifth columns show the strategic value per dollar of put option held by the large bondholder and each of the 20 small bondholders, respectively, calculated using Lemma 4, and the last column shows the total strategic value of the options. The total intrinsic value of the options is \$11 million.

Φ	\bar{v}	E_F	ϕ^{D^1}/D^1	ϕ^D/D	SV
0.0025	7.574	6.866	0.216	0.191	0.096
0.005	7.574	6.865	0.274	0.242	0.125
0.010	7.562	6.781	0.390	0.344	0.183
0.015	7.550	6.696	0.506	0.446	0.240
0.020	7.538	6.612	0.622	0.548	0.298
0.025	7.525	6.527	0.738	0.649	0.355
0.030	7.513	6.443	0.853	0.751	0.412

6.1. Implications of the strategic model for observed settlements

To apply the pricing formulas in Lemma 4 for the value of large and small bondholders' options, I use \bar{v}_{τ_p} , E_F , D^{NP} , D , D^1 , \mathcal{T} , and N , as calibrated above. The *intrinsic value* of the options should have been $(1 - 0.98) \cdot 550$ or about \$11 million. Therefore, the settled amount appears to be about nine times intrinsic value. However, as shown in Table 2, my calculations suggest that the strategic value should have been even greater. The total settlement amount increases from \$96 million to \$412 million as Φ is raised from 0.25% to 3.0%. As also seen in the table, the assumption about Φ has a small effect on \bar{v}_{τ_p} (the size of the pie in a settlement) but a much sharper reduction in the value of the equity holders in the case of an immediate bankruptcy, thus showing the increased threatening power of the puttable bondholders as Φ varies. The strategic values of the bondholders in the model are high because, with a relatively small trigger amount of only \$110 million, the large bondholder and only one small bondholder can credibly threaten the firm. Similarly, five of the 20 smaller bondholders, even without the large bondholder, can threaten, leading to a large number of threatening coalitions. When $\Phi = 0.02$, a number considered reasonable by several empirical studies, the predicted total settlement is about \$298 million. The total pie to be split between the firm and bondholders, $\bar{v}_{\tau_p} - D_F - E_F$ equals \$370 million; therefore, the firm should have ended up with only about \$72 million of the pie, reflecting the extremely stringent trigger and hence the weak bargaining position of the firm.

It was reported in the press that the large bondholder owning a face value of \$100 million was paid about \$51.5 million for his options while the smaller players collectively owning \$450 million were paid about \$46.35 million. Therefore, the large bondholder was paid about five times as much per dollar as smaller bondholders. Because the large bondholder can be in a larger number of threatening combinations (the middle panel of Fig. 2 is relevant to the given distribution of debt), Lemma 4 implies a higher value per dollar for the large bondholder (about 12%), but not five times as much, as was made in the settlement. Comparing the absolute values of the settlement for the case $\Phi = 0.02$ implies that, while the large bondholder came close to achieving his strategic value in the settlement, the smaller bondholders were vastly underpaid. Before leaving this point, I note that even though my calibration is subject to estimation error in the level of the settlement, which depends critically on the exact stock and bond prices and the bankruptcy costs used for calculations, it is robust in the relative payments made to large and small bondholders, which as shown in Section 4.2 only depend on the distribution of debt owned by various bondholders and the trigger value. The model is capable of producing strategic values per dollar five times greater for the large bondholder: indeed, if the amounts of putable debt held by the large and the small bondholders were reversed, the large bondholder would have a per dollar strategic value 12 times higher.

One can turn around and ask what level of Φ will result in the total observed settlement of \$98 million, the observed level? The answer is 0.3%, which is consistent with the view that the puts would cause pure financial distress that could be resolved easily. However, this level of Φ would not be able to match the observed relative payment per dollar, with the observed distribution of putable debt.

One can also ask if there are small changes in the firm's policy that could reduce its vulnerability to putable bondholders. When $\Phi = 0.02$, the model suggests that if there were a single bondholder owning all the debt, the payout for the firm would have been \$100 million higher! In this case, the firm and the bondholder would each get one-half of the pie to be shared. Splitting the putable debt among a larger number of bondholders makes the firm vulnerable to a larger number of threatening coalitions. For instance, if the \$550 million were split between two bondholders, then the firm's share would be reduced to one-third of the pie. Similarly, having a trigger of about \$275 million, or half the total putable debt outstanding, would have saved the firm almost \$120 million at the time of settlement.

In summary, the calibration exercise reveals the following. Both large and small bondholders were likely paid higher than intrinsic value but the large bondholder received a disproportionately larger settlement. The total strategic value of all bondholders equaled four-fifths of the pie to be shared with the firm because a stringent liquidity trigger implied that a large number of credibly

threatening coalitions could be formed by the bondholders. The firm could potentially incur bankruptcy costs on its assets valued at \$16.9 billion, from the put of a relatively small amount of debt of \$550 million, suggesting that the puttable bond issuance was an inefficient capital structure decision. The observed aggregate settlement is consistent with a very small level of financial distress costs. The theoretical model assumes that all the bondholders participate actively in the negotiations. If, instead, small bondholders behave passively as in Dunn and Spatt (1984), then the model predicts that they will not realize their strategic values.

7. Conclusions and extensions

Firms in distress often restructure only a subset of their outstanding debt contracts through private negotiations (after violations of specified covenants in the loan contract) and are thus able to conserve on bankruptcy costs (see, Gilson et al., 1990). I explicitly model such negotiations for the case of puttable securities, which permit bondholders to receive par value under specified events in the option contract when the firm is unable to make the required payments due to either a shortfall of cash or additional restrictive covenants imposed by senior debt holders. It is typical for such cross-provision of covenants to be specified because they limit the extent of the effective violation of seniority through put exercise. Similar put options are embedded in several other lending contracts such as bank loans.

In the bargaining game modeled, puttable bondholders attempt to extract from the firm a fraction of the reduction in bankruptcy costs that would be incurred if the bondholders in fact put their bonds immediately. My analysis reveals that, with this threat, bondholders can extract more than the intrinsic values of their options. Strategic value is determined by the proportion of credibly threatening coalitions that can be formed, and as shown by several examples, this proportion depends not only on the distribution of puttable debt among bondholders but also on the position of the trigger relative to the amounts of debt held by different bondholders. Therefore, covenants can play an important role in the firm's determination of the optimal distribution of puttable debt across lenders. The major analytical results include characterizations of (a) different strategic values per dollar for large and small bondholders, (b) the optimal (from the point of view of the firm) number of puttable bondholders, (c) a simple and easy calculation of the strategic value for bonds held by infinitesimal bondholders, and (d) an *ex ante* calculation of strategic value that permits calibration to credit ratings of the yield reductions for poison puts compared to similar nonputtable bonds. Two calibration exercises provide evidence that strategic behavior is likely an important determinant to prices of poison put bonds and observed settlement amounts. In crisis situations, such as that of

General American Life Insurance, with the puttable debt is held by institutional investors who are not permitted by regulation to strip the debt of its puts and hold the remaining riskier bonds, the firm is either unable to avoid bearing distress costs or the strategic value is extracted by an alternative buyer of the firm.

I see three important extensions that naturally arise from the analysis in this paper. Firstly, the bargaining approach is likely to provide a better understanding of the observed variation in recoveries of different debt claims *in* bankruptcy depending on the firm’s capital structure, the distribution of its debt claims, and the restrictiveness of its covenants (see, e.g., Franks and Torous, 1989). Secondly, it can provide an understanding of the observed restructuring of debt claims (including maturity and coupon changes) that arise as a solution to the renegotiation between shareholders and bondholders after the borrower violates covenants that are set in bond contracts (see, e.g., Smith and Warner, 1979). Thirdly, the analysis can also be extended to the bank loan and private placement markets, in which it is also typical for lenders to impose covenants on the firm’s debt and in which large players of different sizes are prevalent (see, e.g., Carey et al., 1993). A strategic value approach might partly explain why the nature of covenants varies across public bonds, bank loans, and private placements.

Appendix

Proof of Result 1. There are three requirements for the set $\bar{R}_{\tau_p}(\cdot)$

(A.1) For each coalition S , the set $\bar{R}_{\tau_p}(\cdot)$ is closed, convex, and comprehensive (i.e., $\bar{R}_{\tau_p}(\cdot) - \mathcal{R}_+^S \subset \bar{R}_{\tau_p}(\cdot)$). Moreover, $0 \in \bar{R}_{\tau_p}(\cdot)$ and $\bar{R}_{\tau_p}(\cdot) \cap \mathcal{R}_+^S$ is bounded.

(A.2) For each coalition S , the boundary of $\bar{R}_{\tau_p}(\cdot)$ is smooth and nonlevel (i.e., the outward normal vector at any point of the boundary is positive in all coordinates).

(A.3) Monotonicity: $\bar{R}_{\tau_p}(\cdot) \times \{0^{T \setminus S}\} \subset T$ (i.e., if one completes a vector in $\bar{R}_{\tau_p}(\cdot)$ with zeros for the coordinates in $T \setminus S$, then one obtains a vector in $\bar{R}_{\tau_p}(\cdot)(T)$).

For the set $\bar{R}_{\tau_p}(S) = \{c \in \mathcal{R}^S : \sum_{i \in S} c^i \leq v_{\tau_p}(S)\}$, (A.1) and (A.2) trivially hold. For (A.3), it is required that $\bar{R}_{\tau_p}(S) \leq \bar{R}_{\tau_p}(T)$ whenever $S \subset T$. It is evident from the definition of \bar{R}_{τ_p} that the only nonmonotonicities can arise when $D_p = \mathcal{T}$. By Lemma 2, if Condition 1 holds then $\bar{v}_{\tau_p} > E_F + D_F$, and therefore $v_{\tau_p}(E \cup D_C)$ increases as D_C crosses the threshold $(D - \mathcal{T})$, the smallest level of cooperating puttable debt needed to avoid an immediate reorganization. If Condition 1 holds, then $D_F = D$, and therefore, $v_{\tau_p}(D_p)$ increases as D_p crosses the trigger level \mathcal{T} .

Then by Proposition 1 and Theorem 2 in Hart and Mas-Colell (1996), the unique equilibrium payoff vector of the game defined as $\rho \rightarrow 1$ is the Shapley value, and in equilibrium this vector is proposed by the first player and is accepted by all the other players. Because we have assumed that agents are

expected market value maximizers, the value of a coalition S is simply $v_{\tau_p}(S)$, as defined in Eqs. (13) to (17), and the Shapley value of player i is defined in the statement of the Result. \square

Proof of Lemma 3. We will show that the marginal value of player i in any coalition exceeds D_i . Under Condition 1, Eqs. (13) and (14) imply that the value of any pure bondholder coalition with debt D_S equals D_S ; therefore, the marginal contribution of a bondholder with debt D_i to all such coalitions is D_i . Now consider the marginal value of the bondholder in coalitions that include the firm. Let D_C be the total value of debt of cooperating bondholders that includes D_i . If $D_C - D_i \geq (D - \mathcal{F})$, then with or without the cooperation of i , the firm need not file for a reorganization immediately; using Eq. (16), the marginal value of bondholder i equals D_i . The last case is when $D_C - D_i < (D - \mathcal{F})$, but $D_C \geq (D - \mathcal{F})$. In this case, bondholder i is pivotal; using (16) and (17), the marginal value of player i equals

$$\begin{aligned} & v_{\tau_p}[E \cup D_C] - v_{\tau_p}[E \cup (D_C - D_i)] \\ &= \bar{v}_{\tau_p} - (D - D_C) - \left[D_F \frac{D_C - D_i}{D} + E_F \right] \\ &= \bar{v}_{\tau_p} - D_F - E_F + D_i, \end{aligned}$$

and Lemma 2 implies that the last quantity exceeds D_i .

Proof of Result 2. Substituting Eqs. (21)–(24) into Eq. (25) implies

$$\begin{aligned} \phi_{\tau_p}^{E,N} &= \frac{1}{N+1} \left[\sum_{n=0}^{N-M^c} E_F + \frac{n}{N} D_F + \sum_{n=N-M^c+1}^N \bar{v}_{\tau_p} - \frac{N-n}{N} D \right. \\ &\quad \left. - \sum_{n=0}^{M^c-1} \frac{n}{N} D - \sum_{n=M^c}^N \frac{n}{N} D_F \right]. \end{aligned} \tag{41}$$

Collecting terms implies that

$$\phi_{\tau_p}^E = E_F \frac{N - M^c + 1}{N + 1} + \frac{M^c}{N + 1} \left[\bar{v}_{\tau_p} - D \frac{M^c - 1}{N} - D_F \frac{N - M^c + 1}{N} \right]. \tag{42}$$

If Condition 1 holds, then $D_F = D$ and $\phi_{\tau_p}^{E,N}$ is given by Eq. (26). The other results immediately follow.

Lemma 4. The strategic value of the equity holders at τ_p when Condition 1, $P_D < 1$, putable junior debt of D is held by N equally sized bondholders, and D^1 is held by

one large bondholder is given by

$$\begin{aligned}
 \phi_{\tau_F}^{E,N} = & \frac{1}{(N+2)(N+1)} \left[\sum_{m=0}^{N-M^0} \left(\frac{\frac{m}{N}D + D^1}{D + D^1} D_F + E_F \right) (m+1) \right. \\
 & + \sum_{N-M^0+1}^N \left(\bar{v}_{\tau_F} - \left(\frac{N-m}{N} D \right) \right) (m+1) \\
 & - \sum_{m=0}^{M^1-1} \left(\frac{m}{N} D + D^1 \right) (m+1) - \sum_{m=M^1}^N \left(\frac{\frac{m}{N}D + D^1}{D + D^1} D_F \right) (m+1) \\
 & + \sum_{m=0}^{N-M^1} \left(\frac{\frac{m}{N}D}{D + D^1} D_F + E_F \right) (N+1-m) \\
 & + \sum_{N-M^1+1}^N \left(\bar{v}_{\tau_F} - \left(D^1 + \frac{N-m}{N} D \right) \right) (N+1-m) \\
 & - \sum_{m=0}^{M^0-1} \frac{m}{N} D \cdot (N+1-m) \\
 & \left. - \sum_{m=M^0}^N \left(\frac{\frac{m}{N}D}{D + D^1} D_F + E_F \right) (N+1-m) \right]. \tag{43}
 \end{aligned}$$

The strategic value of the large bondholder is given by

$$\begin{aligned}
 \phi_{\tau_F}^{D^1,N} = & \frac{1}{(N+2)(N+1)} \left[\sum_{m=0}^{M^1-1} \left(\frac{m}{N} D + D^1 \right) (N-m+1) \right. \\
 & + \sum_{M^1}^N \left(\frac{\frac{m}{N}D + D^1}{D + D^1} D_F \right) (N-m+1) \\
 & - \sum_{m=0}^{M^0-1} \left(\frac{m}{N} D \right) (N-m+1) - \sum_{m=M^0}^N \left(\frac{\frac{m}{N}D}{D + D^1} D_F \right) (N-m+1) \\
 & + \sum_{m=0}^{N-M^0} \left(\frac{\frac{m}{N}D + D^1}{D + D^1} D_F + E_F \right) (m+1) \\
 & + \sum_{m=N-M^0+1}^N \left(\bar{v}_{\tau_F} - \left(\frac{N-m}{N} D \right) \right) (m+1) \\
 & - \sum_{m=0}^{N-M^1} \left(\frac{\frac{m}{N}D}{D + D^1} D_F + E_F \right) (m+1) \\
 & \left. - \sum_{m=N-M^1+1}^N \left(\bar{v}_{\tau_F} - \left(\frac{N-m}{N} D + D^1 \right) (m+1) \right) \right]. \tag{44}
 \end{aligned}$$

The value of each of the smaller bondholders equals $(\bar{v}_{\tau_F} - \phi^E - \phi^{D^1})/N$.

The summations in the two equations lead to long polynomials that can be easily written down using a symbolic mathematical program such as Mathematica. I have preferred to leave them in the current form because the solutions are more transparent for the reader under this form. Details of the proof are available upon request from the author.

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